Today's Plan:

Learning Target (standard): I will use the 2nd derivative test to describe the characteristics of a function.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Use the 2nd Derivative Test to analyze the function.

$f(x)=12+2x^2-x^4$ D: $\mathbb{R}$

$f'(x)=4x-4x^3$ D: $\mathbb{R}$

$0=4x(1-x^2)$

$0=4x(1+x)(1-x)$

critical #s:

$x=-1,0,1$

$f''(x)=4-12x^2$

$0=4(1-3x^2)$

$0=4(1+\sqrt{3}x)(1-\sqrt{3}x)$

possible POI's:

$x=-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

Domain Interval	4	$1+\sqrt{3}x$	$1-\sqrt{3}x$	$f''(x)$	$f(x)$
$(-\infty,-\frac{\sqrt{3}}{3})$	+	−	+	−	concave down >POI $(-\frac{\sqrt{3}}{3},\frac{113}{9})$
$(-\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3})$	+	+	+	+	concave up >POI $(\frac{\sqrt{3}}{3},\frac{113}{9})$
$(\frac{\sqrt{3}}{3},\infty)$	+	+	−	−	concave down

∴ Since $f''(x)$ goes from negative to positive @ $x=-\frac{\sqrt{3}}{3}$, $f(x)$ goes from concave down to concave up & will have a point of inflection @ $(-\frac{\sqrt{3}}{3},\frac{113}{9})$. Since $f''(x)$ goes from positive to negative @ $x=\frac{\sqrt{3}}{3}$, $f(x)$ goes from concave up to concave down @ $x=\frac{\sqrt{3}}{3}$ and will have a point of inflection @ $(\frac{\sqrt{3}}{3},\frac{113}{9})$.

$f(-\frac{\sqrt{3}}{3})=12+2(-\frac{\sqrt{3}}{3})^2-(-\frac{\sqrt{3}}{3})^4$

$=12+\frac{2}{3}-\frac{1}{9}$

$=\frac{108}{9}+\frac{6}{9}-\frac{1}{9}$

$f(-\frac{\sqrt{3}}{3})=\frac{113}{9}$

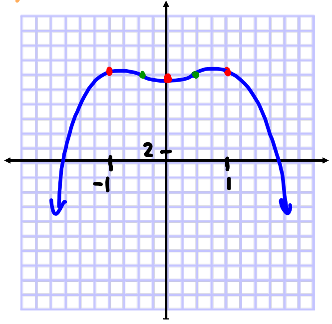
Extremum:

$f''(-1)=4-12(-1)^2=4-12=-8<0$ max $=13$ @ $x=-1$

$f''(0)=4-12(0)=4>0$ min $=12$ @ $x=0$

$f''(1)=4-12(1)^2=4-12=-8<0$ max $=13$ @ $x=-1$

∴ Since $f''(-1)$ is negative, $f(x)$ will have a maximum of 13 @ $x=-1$. Since $f''(0)$ is positive, $f(x)$ will have a minimum of 12 @ $x=0$. Since $f''(1)$ is negative, $f(x)$ will have a maximum of 13 @ $x=1$.



Use the 2nd Derivative Test to analyze the function.

$$f(x)=\frac{x}{x^2+1} \quad D:\mathbb{R}$$

$$f'(x)=\frac{1(x^2+1)-2x(x)}{(x^2+1)^2}$$

$$=\frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$f'(x)=\frac{1-x^2}{(x^2+1)^2}$$

$$0=(1+x)(1-x)$$

Critical #s:
$x=-1,1$

$$f''(x)=\frac{-2x(x^2+1)^2-2(x^2+1)(2x)(1-x^2)}{(x^2+1)^4}$$

$$=\frac{-2x(x^2+1)^2-4x(x^2+1)(1-x^2)}{(x^2+1)^4}$$

$$=\frac{-2x(x^2+1)\left[x^2+1+2(1-x^2)\right]}{(x^2+1)^4}$$

$$=\frac{-2x(x^2+1)(x^2+1+2-2x^2)}{(x^2+1)^4}$$

$$=\frac{-2x(-x^2+3)}{(x^2+1)^3}$$

$$f''(x)=\frac{2x(x^2-3)}{(x^2+1)^3}$$

$$0=2x(x+\sqrt{3})(x-\sqrt{3})$$

Possible POI's: $x=-\sqrt{3},0,\sqrt{3}$

Domain Interval	$2x$	x^2-3	$(x^2+1)^3$	$f''(x)$	Concavity of $f(x)$
$(-\infty,-\sqrt{3})$	−	+	+	−	down
					> POI $\left(-\sqrt{3},\frac{\sqrt{3}}{4}\right)$
$(-\sqrt{3},0)$	−	−	+	+	up
					> POI $(0,0)$
$(0,\sqrt{3})$	+	−	+	−	down
					> POI $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$
$(\sqrt{3},\infty)$	+	+	+	+	up

Extrema:

$f''(-1)=\frac{-2(-2)}{8}=\frac{4}{8}>0$ min $=-\frac{1}{2}$ @ $x=-1$

$f''(1)=\frac{2(-2)}{8}=-\frac{4}{8}<0$ max $=\frac{1}{2}$ @ $x=1$

End Behavior:

$$\lim_{x\to\infty}\frac{x}{x^2+1}=\lim_{x\to\infty}\frac{\frac{x}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{1+\frac{1}{x^2}}=\frac{0}{1}=0$$

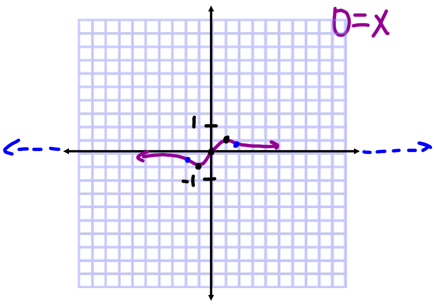
$$\lim_{x\to-\infty}\frac{\frac{1}{x}}{1+\frac{1}{x^2}}=\frac{0}{1}=0$$

$\therefore$ HA: $y=0$

intersects? @ $(0,0)$

$$0=\frac{x}{x^2+1}$$

$$0=x$$



Assignment:

p.159 #5, 7, 8

rational function

* Choose one to write the explanation of the 2nd derivative test *