Today's Plan:

Learning Target (standard): I will find the antiderivative of a function.

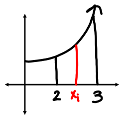
Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Find the area under $y = x^2 + 1$ on the interval [2,3].



$\Delta x = \frac{b-a}{n} = \frac{3-2}{n} = \frac{1}{n}$

$x_0 = 2$

$x_1 = 2 + \Delta x = 2 + \frac{1}{n}$

$x_2 = 2 + 2\Delta x = 2 + \frac{2}{n}$

$\vdots$

$x_i = 2 + i\Delta x = 2 + \frac{i}{n}$

$\vdots$

$x_n = 2 + n\Delta x = 2 + n\left(\frac{1}{n}\right)$

$= 2 + 1$

$= 3 \checkmark$

$A_{R_i} = \Delta x \cdot f(x_i)$

$= \frac{1}{n}\left[\left(2+\frac{i}{n}\right)^2 + 1\right]$

$= \frac{1}{n}\left[4 + \frac{4i}{n} + \frac{i^2}{n^2} + 1\right]$

$= \frac{1}{n}\left[5 + \frac{4i}{n} + \frac{i^2}{n^2}\right]$

$A_{R_i} = \frac{5}{n} + \frac{4i}{n^2} + \frac{i^2}{n^3}$

$A = \sum_{i=1}^{n}\left(\frac{5}{n} + \frac{4i}{n^2} + \frac{i^2}{n^3}\right)$

$= \sum_{i=1}^{n}\frac{5}{n} + \sum_{i=1}^{n}\frac{4i}{n^2} + \sum_{i=1}^{n}\frac{i^2}{n^3}$

$= \frac{5}{n}(n) + \frac{4}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$

$2n^2+3n+1$

$= 5 + \frac{\cancel{4}^{2}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}_{1}}\right) + \frac{1}{n^{\cancel{3}2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)$

$= 5 + \frac{2}{n}(n+1) + \frac{1}{6n^2}\left(2n^2+3n+1\right)$

$= 5 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$

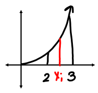
$= \frac{22}{3} + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2}$

$A = \lim_{n\to\infty}\left(\frac{22}{3} + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2}\right)$

$= \frac{22}{3} + 0 + 0 + 0$

$A = \frac{22}{3}u^2$

Find the area under f(x) = x^2 on the interval [2,3]. Write as a definite integral. Use this to evaluate $\int_2^3 (x^2+1)\,dx$.



$\Delta x = \frac{b-a}{n} = \frac{3-2}{n} = \frac{1}{n}$

$x_0 = 2$

$x_1 = 2 + \Delta x = 2 + \frac{1}{n}$

$x_2 = 2 + 2\Delta x = 2 + \frac{2}{n}$

$\vdots$

$x_i = 2 + i\Delta x = 2 + \frac{i}{n}$

$\vdots$

$x_n = 2 + n\Delta x = 2 + n\left(\frac{1}{n}\right) = 2 + 1 = 3$ ✓

$A_{R_i} = \Delta x \cdot f(x_i) = \frac{1}{n}\left[\left(2+\frac{i}{n}\right)^2\right] = \frac{1}{n}\left(4 + \frac{4i}{n} + \frac{i^2}{n^2}\right)$

$A_{R_i} = \frac{4}{n} + \frac{4i}{n^2} + \frac{i^2}{n^3}$

$$A = \sum_{i=1}^{n} A_{R_i}$$

$$= \sum_{i=1}^{n}\left(\frac{4}{n} + \frac{4i}{n^2} + \frac{i^2}{n^3}\right)$$

$$= \sum_{i=1}^{n}\frac{4}{n} + \sum_{i=1}^{n}\frac{4i}{n^2} + \sum_{i=1}^{n}\frac{i^2}{n^3}$$

$$= \frac{4}{n}(n) + \frac{4}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= 4 + \frac{\cancel{4}^{2}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}_{1}}\right) + \frac{1}{n^{\cancel{3}\,2}}\left(\frac{\cancel{n}\overbrace{(n+1)(2n+1)}^{2n^2+3n+1}}{6}\right)$$

$$= 4 + \frac{2}{n}(n+1) + \frac{1}{6n^2}(2n^2+3n+1)$$

$$= 4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

$$= \frac{19}{3} + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2}$$

$$A = \lim_{n\to\infty}\left(\frac{19}{3} + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2}\right)$$

$$= \frac{19}{3} + 0 + 0 + 0$$

$A = \frac{19}{3}u^2$ $= \int_2^3 x^2\,dx$

$$\int_2^3 (x^2+1)\,dx = \int_2^3 x^2\,dx + \int_2^3 dx$$

$$= \frac{19}{3} + 1(3-2)$$

$$= \frac{19}{3} + 1$$

$$= \frac{22}{3}$$

$$\boxed{\int_0^2 x^3\,dx = 4}$$

$$\boxed{\int_0^2 x\,dx = 2}$$

$$\int_0^2 \left(3x^3 - 4x + 5\right)dx =$$

$$= \int_0^2 3x^3\,dx - \int_0^2 4x\,dx + \int_0^2 5\,dx$$

$$= 3\boxed{\int_0^2 x^3\,dx} - 4\boxed{\int_0^2 x\,dx} + \int_0^2 5\,dx$$

$$= 3(4) - 4(2) + 5(2-0)$$

$$= 12 - 8 + 10$$

$$= 14$$

$$\boxed{\int_1^4 \sqrt{x}\,dx = \frac{14}{3}}$$

$$\int_1^4 \left(\sqrt{10x}+3\right)dx = \int_1^4 \left(\sqrt{10}\cdot\sqrt{x}\right)dx + \int_1^4 3\,dx$$

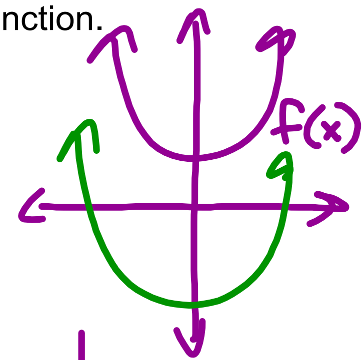
$$= \sqrt{10}\boxed{\int_1^4 \sqrt{x}\,dx} + \int_1^4 3\,dx$$

$$= \sqrt{10}\left(\frac{14}{3}\right) + 3(4-1)$$

$$= \frac{14\sqrt{10}}{3} + 9$$

$$= \frac{14\sqrt{10}+27}{3}$$

Find the function that has as its derivative the given function.



$$f'(x) = 2x - 5$$

$$f(x) = x^2 - 5x + \pi$$

constant

$$f(x) = x^2 - 5x + C$$

Find the function that has as its derivative the given function.

$$f'(x) = \frac{1}{4}x^2 + 3x - 1$$

$$f(x) = \frac{1}{12}x^3 + \frac{3}{2}x^2 - x + \sqrt{3}$$ "constant"

$$f(x) = \frac{1}{12}x^3 + \frac{3}{2}x^2 - x + C$$

Find the function that has as its derivative the given function.

$$f'(x) = 4x^3 - 5x^2 + \frac{2}{7}x - 3$$

$$f(x) = 4x^3 - 5x^2 + \frac{2}{7}x - 3$$

$$\cancel{f(x)} = x^4 - \frac{5}{3}x^3 + \frac{1}{7}x^2 - 3x + C$$

$F(x)$

$F(x) =$

Now use your formula to find the function that has the given derivative.

$$f'(x) = x^{\frac{4}{3}} + x^{\frac{7}{2}} + \sqrt{x}$$

$x^{\frac{1}{2}}$

$$f(x) = \frac{3}{7}x^{\frac{7}{3}} + \frac{2}{9}x^{\frac{9}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$$

$$f(x) = \frac{3}{7}x^{\frac{7}{3}} + \frac{2}{9}x^{\frac{9}{2}} + \frac{2}{3}\sqrt{x^3} + C$$

Antiderivatives:

Power Rule:

$$f(x) = ax^r$$ "the derivative"

$$F(x) = \left(\frac{a}{r+1}\right)x^{r+1} + c$$

"antiderivative"

Antiderivatives:

- Derivatives ask the question "Given the function *f(x)*, find the derivative *f ' (x)".*

- Antiderivatives ask the question "Given the derivative *f ' (x)*, find the function *f(x)".*

Antiderivatives:

- A function *F(x)* is an antiderivative of a function *f(x)* if *F'(x)* = *f(x)*.

- Antiderivatives are never unique and give us a family of functions that has as its derivative the antiderivative. Why?

$$y' = f(x) = 8x^3$$
$$F(x) = 2x^4 + C$$

Find *F(x)*.

$$f(x) = 4x^5$$

$$F(x) = \frac{2}{3}x^6 + C$$

Find *F(x)*.

$$f(x) = 3x^4 - x + 4 + \frac{5}{x^3}$$

$$f(x) = 3x^4 - x + 4 + 5x^{-3}$$

$$F(x) = \frac{3}{5}x^5 - \frac{1}{2}x^2 + 4x - \frac{5}{2}x^{-2} + C$$

$$F(x) = \frac{3}{5}x^5 - \frac{1}{2}x^2 + 4x - \frac{5}{2x^2} + C$$

Assignment:

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