# Today's Plan:

***Learning Target (standard)***: I will estimate the area under a curve using Riemann sums.

***Students will***: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

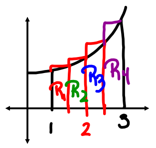
***Teacher will***: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

***Assessment***: Board work, homework check and homework assignment

***Differentiation***: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Estimate the area under $y = x^2 + 2$ on the interval [1,3].

Use 4 right-endpoint rectangles.




$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

$$A_{R_1} = \frac{1}{2}\left(f\left(\frac{3}{2}\right)\right)$$
$$= \frac{1}{2}\left(\frac{9}{4}+2\right)$$
$$= \frac{1}{2}\left(\frac{9}{4}+\frac{8}{4}\right)$$
$$A_{R_1} = \frac{17}{8}u^2$$

$$A_{R_2} = \frac{1}{2}(f(2))$$
$$= \frac{1}{2}(4+2)$$
$$A_{R_2} = 3u^2$$

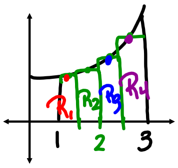
$$A_{R_3} = \frac{1}{2}\left(f\left(\frac{5}{2}\right)\right)$$
$$= \frac{1}{2}\left(\frac{25}{4}+2\right)$$
$$= \frac{1}{2}\left(\frac{25}{4}+\frac{8}{4}\right)$$
$$A_{R_3} = \frac{33}{8}u^2$$

$$A_{R_4} = \frac{1}{2}(f(3))$$
$$= \frac{1}{2}(9+2)$$
$$A_{R_4} = \frac{11}{2}u^2$$

$$A \approx \frac{17}{8} + 3 + \frac{33}{8} + \frac{11}{2}$$
$$\approx \frac{17+24+33+44}{8}$$
$$\approx \frac{118}{8}$$
$$A \approx \frac{59}{4}u^2$$

Estimate the area under $y = x^2 + 2$ on the interval [1,3].

Use 4 mid-points.




$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$

$A_{R_1} = \frac{1}{2}\left(f\left(\frac{5}{4}\right)\right)$

$= \frac{1}{2}\left(\frac{25}{16} + \frac{32}{16}\right)$

$= \frac{1}{2}\left(\frac{57}{16}\right)$

$A_{R_1} = \frac{57}{32} u^2$

$A_{R_2} = \frac{1}{2}\left(f\left(\frac{7}{4}\right)\right)$

$= \frac{1}{2}\left(\frac{49}{16} + \frac{32}{16}\right)$

$A_{R_2} = \frac{81}{32} u^2$

$A_{R_3} = \frac{1}{2}\left(f\left(\frac{9}{4}\right)\right)$

$= \frac{1}{2}\left(\frac{81}{16} + \frac{32}{16}\right)$

$A_{R_3} = \frac{113}{32} u^2$

$A_{R_4} = \frac{1}{2}\left(f\left(\frac{11}{4}\right)\right)$

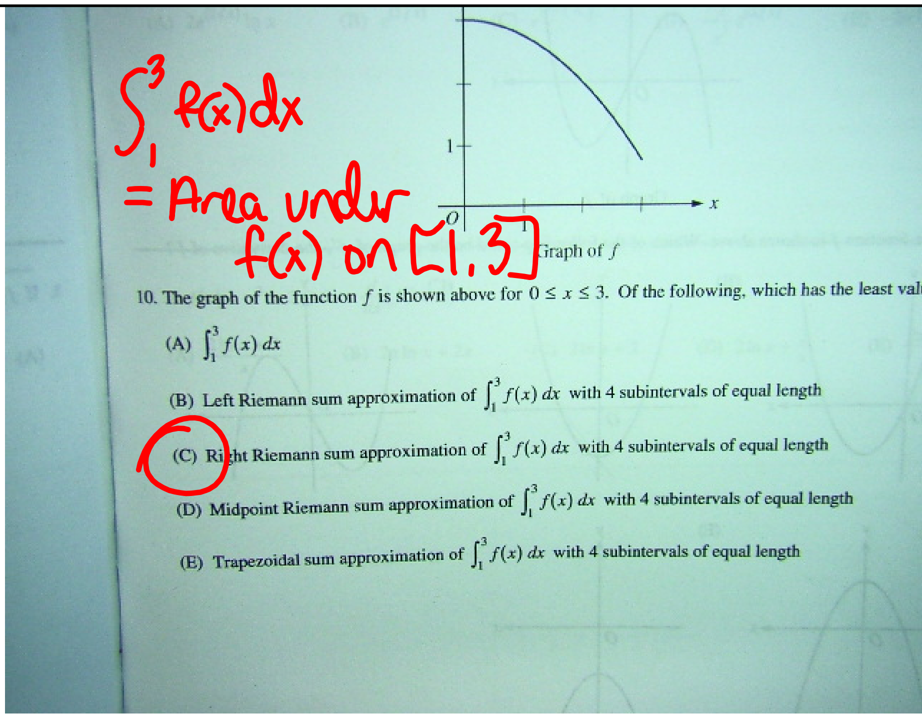
$= \frac{1}{2}\left(\frac{121}{16} + \frac{32}{16}\right)$

$A = \frac{153}{32} u^2$

$A \approx \frac{57}{32} + \frac{81}{32} + \frac{113}{32} + \frac{153}{32}$

$\approx \frac{404}{32}$

$A \approx \frac{101}{8} u^2$



$\int_1^3 f(x)\,dx$

= Area under f(x) on [1,3]

Graph of $f$

10. The graph of the function $f$ is shown above for $0 \le x \le 3$. Of the following, which has the least val

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

C is the correct choice.

Find the sum:

4 - ending point

$$\sum_{i=1}^{4} i^2(i-3)$$

function

"index"
-starting point

"the sum from i = 1 to 4 of $i^2(i-3)$"

"sigma" - summation

$= 1^2(1-3) + 2^2(2-3) + 3^2(3-3) + 4^2(4-3)$

$= -2 - 4 + 0 + 16$

$= 10$

Find the sum:

$$\sum_{i=1}^{5} 10 = 10 + 10 + 10 + 10 + 10$$

$= 50$

## Sum Formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{4} i = 1+2+3+4 = \frac{4(4+1)}{2} = \frac{4(5)}{2}$$

$$= 10$$

$$= \frac{20}{2} = 10$$

## Sum Formulas:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{4} i^2 = 1+4+9+16 = \frac{4(4+1)(8+1)}{6}$$

$$= 30$$

$$= \frac{4(5)(9)}{6} = \frac{180}{6}$$

$$= 30$$

# Sum Formulas:

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{4} i^3 = 1+8+27+64$$
$$= 100$$

$$= \left[\frac{4(4+1)}{2}\right]^2$$
$$= \left[\frac{4(5)}{2}\right]^2$$
$$= 10^2 = 100$$

# Properties of Summations:

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} c a_i = c\sum_{i=1}^{n} a_i$$

# Assignment:

p.214 #1-10

#17 - set up to area of rectangle