

Today's Plan:

Learning Target (standard): I will evaluate definite integrals using the Fundamental Theorem of Calculus.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

p.201 #2-24 even

$$2) F(x) = \frac{4}{3}x^3 - 4x^2 + x + C$$

$$4) F(x) = 2x^5 - \frac{3}{2}x^4 + 5x + C$$

$$6) F(x) = -\frac{2}{3x^6} + \frac{7}{3x^3} + \frac{1}{2}x^2 + C$$

$$8) F(x) = \frac{2}{5}\sqrt{x^5} + \frac{1}{2}x^{-1} + 5x + C$$

$$10) F(x) = \frac{1}{2}x^6 - \frac{3}{8}\sqrt[3]{x^8} + C$$

$$12) F(x) = \frac{1}{3}x^3 - 2x - \frac{1}{x} + C$$

$$14) F(x) = 2x^3 - \frac{13}{2}x^2 - 5x + C$$

$$16) F(x) = \frac{4}{5}\sqrt{x^5} - \frac{2}{3}\sqrt{x^3} + 6\sqrt{x} + C$$

$$18) F(x) = \frac{3}{2}\sqrt[3]{x^8} + C$$

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$$20) F(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + x + C$$

$$22) f(x) = 3x^3 + \frac{1}{2}x^2 - 8x - \frac{9}{2}$$

$$24) f''(x) = 3x^2 + 2$$

$$f'(x) = x^3 + 2x - 1$$

$$f(x) = \frac{1}{4}x^4 + x^2 - x + 4$$

Use a summations to find the area under $f(x) = x^2 - 1$ on $[1, 3]$.

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$
 $x_0 = 1$
 $x_1 = 1 + \Delta x = 1 + \frac{2}{n}$
 $x_2 = 1 + 2\Delta x = 1 + \frac{4}{n}$
 \vdots
 $x_i = 1 + i\Delta x = 1 + \frac{2i}{n}$
 \vdots
 $x_n = 1 + n\Delta x = 1 + n\left(\frac{2}{n}\right) = 1 + 2 = 3$

$A_{R_i} = \Delta x \cdot f(x_i)$
 $= \frac{2}{n} \left[\left(1 + \frac{2i}{n}\right)^2 - 1 \right]$
 $= \frac{2}{n} \left[1 + \frac{4i}{n} + \frac{4i^2}{n^2} - 1 \right]$
 $= \frac{2}{n} \left(\frac{4i}{n} + \frac{4i^2}{n^2} \right)$
 $A_{R_i} = \frac{8i}{n^2} + \frac{8i^2}{n^3}$

$A = \sum_{i=1}^n A_{R_i} = \sum_{i=1}^n \left(\frac{8i}{n^2} + \frac{8i^2}{n^3} \right)$
 $= \sum_{i=1}^n \frac{8i}{n^2} + \sum_{i=1}^n \frac{8i^2}{n^3}$
 $= \frac{8}{n^2} \sum_{i=1}^n i + \frac{8}{n^3} \sum_{i=1}^n i^2$
 $= \frac{8}{n^2} \left(\frac{n(n+1)}{2} \right) + \frac{8}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right)$
 $= \frac{4}{n} (n+1) + \frac{4}{3n^2} (2n^2 + 3n + 1)$
 $= 4 + \frac{4}{n} + \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$
 $= \frac{20}{3} + \frac{8}{n} + \frac{4}{3n^2}$

$A = \lim_{n \rightarrow \infty} \left(\frac{20}{3} + \frac{8}{n} + \frac{4}{3n^2} \right)$
 $= \frac{20}{3} + 0 + 0$
 $A = \frac{20}{3} \text{ u}^2$

Find the antiderivative.

$$f(x) = \frac{6}{\sqrt[3]{x}} - \frac{\sqrt[3]{x}}{6} + 7$$

$$f(x) = 6x^{-\frac{1}{3}} - \frac{1}{6}x^{\frac{1}{3}} + 7$$

$$\begin{array}{l} \frac{1}{6} \div \frac{4}{3} \\ \frac{1}{6} \cdot \frac{3}{4} \end{array}$$

$$F(x) = 9x^{\frac{2}{3}} - \frac{1}{8}x^{\frac{4}{3}} + 7x + C$$

$$F(x) = 9\sqrt[3]{x^2} - \frac{\sqrt[3]{x^4}}{8} + 7x + C$$

Find the antiderivative.

$$f(x) = 2x^{\frac{5}{4}} + 6x^{\frac{1}{4}} + 3x^{-4}$$

$$F(x) = \frac{8}{9}x^{\frac{9}{4}} + \frac{24}{5}x^{\frac{5}{4}} - x^{-3} + C$$

Find the antiderivative.

$$f(x) = (6x - 5)^2$$

$$f(x) = 36x^2 - 60x + 25$$

$$F(x) = 12x^3 - 30x^2 + 25x + C$$

Find the antiderivative.

$$f(x) = \frac{8x - 5}{\sqrt[3]{x}}$$

$$f(x) = x^{-\frac{1}{3}}(8x - 5)$$

$$f(x) = 8x^{\frac{2}{3}} - 5x^{-\frac{1}{3}}$$

$$F(x) = \frac{24}{5}x^{\frac{5}{3}} - \frac{15}{2}x^{\frac{2}{3}} + C$$

$$F(x) = \frac{24\sqrt[3]{x^5}}{5} - \frac{15\sqrt[3]{x^2}}{2} + C$$

Find the antiderivative.

$$f(x) = \sqrt[5]{32x^4}$$

$$\sqrt[5]{32} = 2$$

$$f(x) = 2x^{\frac{4}{5}}$$

$$F(x) = \frac{10}{9} x^{\frac{9}{5}} + C$$

$$F(x) = \frac{10}{9} \sqrt[5]{x^9} + C$$

Find the antiderivative.

$$f(x) = \frac{x^3 - 1}{x - 1}$$

$$f(x) = \frac{(x-1)(x^2+x+1)}{x-1}$$

$$f(x) = x^2 + x + 1$$

$$F(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$$

Solve the differential equation with the given condition.

$$f'(x) = 12x^2 - 6x + 1$$

$$f(1) = 5$$

$$x = 1$$

$$y = 5$$

$$f(x) = 4x^3 - 3x^2 + x + C$$

$$5 = 4(1)^3 - 3(1)^2 + 1 + C$$

$$5 = 4 - 3 + 1 + C$$

$$5 = 2 + C$$

$$C = 3$$

$$f(x) = 4x^3 - 3x^2 + x + 3$$

Solve the differential equation with the given condition.

$$f''(x) = 4x - 1$$

$$f'(2) = -2$$

$$f(1) = 3$$

$$f'(x) = 2x^2 - x + C$$

$$-2 = 2(2)^2 - 2 + C$$

$$-2 = 8 - 2 + C$$

$$-2 = 6 + C$$

$$C = -8 \quad f'(x) = 2x^2 - x - 8$$

$$f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 8x + C$$

$$3 = \frac{2}{3}(1)^3 - \frac{1}{2}(1)^2 - 8(1) + C$$

$$3 = \frac{2}{3} - \frac{1}{2} - 8 + C$$

$$11 = \frac{4}{6} - \frac{3}{6} + C$$

$$11 = \frac{1}{6} + C$$

$$C = \frac{65}{6} \quad f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 8x + \frac{65}{6}$$

The Fundamental Theorem of Calculus:

Suppose $f(x)$ is a continuous function on $[a, b]$.

- If the function $F(x)$ is defined by $F(x) = \int_a^x f(t) dt$ for all x in $[a, b]$, then $F(x)$ is an antiderivative of $f(x)$ on $[a, b]$.

- If $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

↑ boundaries ↑ derivative ↑ antiderivative

* definite integrals
= numerical value

$$\int_2^4 5 dx = 5(4-2)$$

$$= 5(2)$$

$$= 10$$

$$f(x) = 5$$

$$F(x) = 5x + C$$

$$\int_2^4 5 dx = 5x + C \Big|_2^4$$

$$= [5(4) + C] - [5(2) + C]$$

$$= 20 + C - 10 - C$$

$$= 10$$

Evaluate (integrate)

$$\int_1^3 (x^2 - 1) dx = \left(\frac{1}{3}x^3 - x \right) \Big|_1^3 = F(b) - F(a)$$

$$= \left[\frac{1}{3}(3)^3 - 3 \right] - \left[\frac{1}{3}(1)^3 - 1 \right]$$

$$= (9 - 3) - \left(\frac{1}{3} - 1 \right)$$

$$= 6 + \frac{2}{3}$$

$$= \frac{20}{3}$$

Assignment:

p.237 #2-22 even