Today's Plan:

Learning Target (standard): I will find the average value of a function and describe its meaning.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Answers to Assignment:

1) $f(c)=\frac{8}{3}$

2) $f(c)=\frac{2\sqrt{2}}{3}$

3) $\sin^2 x$

4) $27x^2-9x$

5) $2x^3$

6) $-2\cos x$

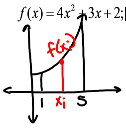
7a) $\frac{91}{2}u^2$

b) $\frac{171}{2}u^2$

c) $\frac{131}{2}u^2$

Find the area using summations. Verify with an integral.

$f(x) = 4x^2 + 3x + 2; [1,5]$



$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

$x_0 = 1$

$x_1 = 1 + \Delta x = 1 + \frac{4}{n}$

$x_2 = 1 + 2\Delta x = 1 + \frac{8}{n}$

$\vdots$

$x_i = 1 + i\,\Delta x = 1 + \frac{4i}{n}$

$\vdots$

$x_n = 1 + n\Delta x = 1 + n\left(\frac{4}{n}\right)$
$= 1 + 4$
$= 5 \checkmark$

$$A_{R_i} = \Delta x \cdot f(x_i)$$
$$= \frac{4}{n}\left[4\left(1+\frac{4i}{n}\right)^2 + 3\left(1+\frac{4i}{n}\right) + 2\right]$$
$$= \frac{4}{n}\left[4\left(1+\frac{8i}{n}+\frac{16i^2}{n^2}\right) + 3 + \frac{12i}{n} + 2\right]$$
$$= \frac{4}{n}\left(4 + \frac{32i}{n} + \frac{64i^2}{n^2} + 5 + \frac{12i}{n}\right)$$
$$= \frac{4}{n}\left(9 + \frac{44i}{n} + \frac{64i^2}{n^2}\right)$$
$$A_{R_i} = \frac{36}{n} + \frac{176i}{n^2} + \frac{256i^2}{n^3}$$

$$A = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n}\left(\frac{36}{n} + \frac{176i}{n^2} + \frac{256i^2}{n^3}\right)$$
$$= \sum_{i=1}^{n}\frac{36}{n} + \sum_{i=1}^{n}\frac{176i}{n^2} + \sum_{i=1}^{n}\frac{256i^2}{n^3}$$
$$= \frac{36}{n}(n) + \frac{176}{n^2}\sum_{i=1}^{n} i + \frac{256}{n^3}\sum_{i=1}^{n} i^2$$
$$= 36 + \frac{\cancel{176}^{\,88}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}\,1}\right) + \frac{\cancel{256}^{\,128}}{n^{\cancel{3}\,2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}\right) \qquad 2n^2+3n+1$$
$$= 36 + \frac{88}{n}(n+1) + \frac{128}{3n^2}(2n^2+3n+1)$$
$$= 36 + 88 + \frac{88}{n} + \frac{256}{3} + \frac{128}{3n} + \frac{128}{3n^2}$$
$$= \frac{628}{3} + \frac{88}{n} + \frac{128}{3n} + \frac{128}{3n^2}$$

$$A = \lim_{n\to\infty}\left(\frac{628}{3} + \frac{88}{n} + \frac{128}{3n} + \frac{128}{3n^2}\right)$$
$$= \frac{628}{3} + 0 + 0 + 0$$
$$A = \frac{628}{3}\,u^2$$

Area under the curve:

$$\int_1^5 \left(4x^2 + 3x + 2\right)dx = \left(\frac{4}{3}x^3 + \frac{3}{2}x^2 + 2x\right)\Big|_1^5$$
$$= \left[\frac{4}{3}(5)^3 + \frac{3}{2}(5)^2 + 2(5)\right] - \left[\frac{4}{3}(1)^3 + \frac{3}{2}(1)^2 + 2(1)\right]$$
$$= \frac{500}{3} + \frac{75}{2} + 10 - \frac{4}{3} - \frac{3}{2} - 2$$
$$= 8 + \frac{496}{3} + \frac{72}{2}$$
$$= 8 + \frac{496}{3} + 36$$
$$= 44 + \frac{496}{3}$$
$$= \frac{132 + 496}{3}$$
$$A = \frac{628}{3}\,u^2$$

Find the integral.

$$\int \frac{x^2+x-1}{x^4}\,dx = \int x^{-4}(x^2+x-1)\,dx$$

$$= \int (x^{-2}+x^{-3}-x^{-4})\,dx$$

$$= -x^{-1} - \frac{1}{2}x^{-2} + \frac{1}{3}x^{-3} + C$$

$$= -\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + C$$

Find the integral.

$$\int (2x+5)(x^2+5x)^7\,dx$$

$u = x^2+5x$

$du = (2x+5)\,dx$

$$\Rightarrow \int u^7\,du$$

$$= \frac{1}{8}u^8 + C$$

$$\Rightarrow \frac{1}{8}(x^2+5x)^8 + C$$

Find the integral.

$$\int \sqrt{x}\left(x^3+\sqrt{x}\right)dx = \int x^{\frac{1}{2}}\left(x^3+x^{\frac{1}{2}}\right)dx$$

$$= \int \left(x^{\frac{7}{2}}+x\right)dx$$

$$= \frac{2}{9}x^{\frac{9}{2}}+\frac{1}{2}x^2+C$$

$$= \frac{2}{9}\sqrt{x^9}+\frac{1}{2}x^2+C$$

Find the integral.

$$\int \frac{x}{\sqrt{9-x^2}}dx \Rightarrow -\frac{1}{2}\int u^{-\frac{1}{2}}du$$

$u=9-x^2$

$du=-2x\,dx$

$-\frac{1}{2}du=x\,dx$

$$=-\frac{1}{2}\left(2u^{\frac{1}{2}}\right)+C$$

$$=-u^{\frac{1}{2}}+C$$

$$\Rightarrow -\sqrt{9-x^2}+C$$

Find the derivative.

$$\frac{d}{dx}\int_1^{x^2}\frac{dt}{1-\sqrt[3]{t}} = \frac{1}{1-\sqrt[3]{x^2}}\cdot 2x$$

$$= \frac{2x}{1-\sqrt[3]{x^2}}$$

Find the derivative.

$$\frac{d}{dx}\int_1^{x^2}\frac{t\,dt}{\sin t} = \frac{x^2}{\sin x^2}\cdot 2x$$

$$= \frac{2x^3}{\sin x^2}$$

Find the average value of $f(x) = x^2$ on [3,6]. Explain the meaning of this.

$$f(c) = \frac{1}{6-3}\int_3^6 x^2\,dx$$

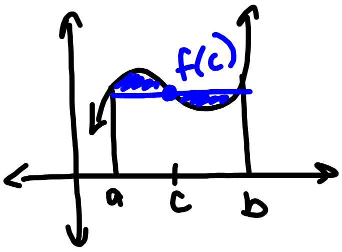
$$= \frac{1}{3}\int_3^6 x^2\,dx$$

$$= \frac{1}{3}\left(\frac{1}{3}x^3\right)\Big|_3^6$$

$$= \frac{1}{9}\left(6^3 - 3^3\right)$$

$$= \frac{1}{9}(216 - 27)$$

$$= \frac{1}{9}(189)$$



$$\boxed{f(c) = 21}$$

The area under f(x) = x^2 on [3,6] will be equal to the area of the rectangle with length (6-3) and height f(c) = 21. So, the area will be 63 u^2.

Evaluate.

$$\int_2^4 \sqrt{x^2 - 2x}\,dx$$

$U = x^2 - 2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x = 4$
$U = 16 - 2$
$U = 14$

$x = 2$
$U = 4 - 2$
$U = 2$

$$\Rightarrow \frac{1}{2}\int_2^{14} u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_2^{14}$$

$$= \frac{1}{3}\left(14^{\frac{3}{2}} - 2^{\frac{3}{2}}\right)$$

$$= \frac{1}{3}\left(14\sqrt{14} - 2\sqrt{2}\right)$$

Find the derivative.

$$\frac{d}{dx}\int_{3x}^{x^2}\left(1-t^2\right)dt$$

$$\int_{3x}^{x^2}(1-t^2)\,dt = \left(t-\tfrac{1}{3}t^3\right)\Big|_{3x}^{x^2}$$

$$= \left(x^2-\tfrac{1}{3}(x^2)^3\right)-\left(3x-\tfrac{1}{3}(3x)^3\right)$$

$$= x^2-\tfrac{1}{3}x^6-3x+\tfrac{1}{3}(27x^3)$$

$$= x^2-\tfrac{1}{3}x^6-3x+9x^3$$

$$\frac{d}{dx}\left(x^2-\tfrac{1}{3}x^6-3x+9x^3\right)$$

$$=2x-2x^5-3+27x^2$$

$$\frac{d}{dx}\int_{3x}^{x^2}(1-t^2)\,dt = \left[1-(x^2)^2\right]\cdot 2x-\left[1-(3x)^2\right]\cdot 3$$

$$=2x(1-x^4)-3(1-9x^2)$$

$$=2x-2x^5-3+27x^2$$

Assignment:

Integration Practice

#1-14