Today's Plan:

Learning Target (standard): I will review integration.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Integration Practice

1) $\frac{4}{3}\sqrt{\left(3x^5+5\right)^3}+C$

2) $-\frac{2}{3}\left(x^2+5\right)^{-3}+C$

3) $\frac{3}{4}\left(2x^2+3\right)^{\frac{8}{3}}+C$

4) $3\left(3x^3-5\right)^{\frac{4}{3}}+C$

5) $\frac{3}{2}\sqrt[3]{\left(3x^4-2\right)^4}+C$

6) $3\sqrt[3]{\left(5x^2+3\right)^4}+C$

7) $\frac{6}{7}\left(x^4+2\right)^{\frac{7}{2}}+C$

8) $\frac{8}{3}\sqrt{\left(3x^5-4\right)^3}+C$

9) $-\frac{1}{\left(4x^4-5\right)^4}+C$

10) $-\frac{1}{\left(x^3+5\right)^2}+C$

11) $f(c)=-2$

12) $f(c)=2$

13) $f(c)=\frac{7}{6}$

14) $f(c)=6$

Evaluate.

$$\int_1^9 2x\sqrt{x}dx = \int_1^9 2x\left(x^{\frac{1}{2}}\right)dx$$

$$= \int_1^9 2x^{\frac{3}{2}}\,dx$$

$$= \frac{4}{5}x^{\frac{5}{2}}\Big|_1^9$$

$$= \frac{4}{5}\left(9^{\frac{5}{2}} - 1^{\frac{5}{2}}\right)$$

$$= \frac{4}{5}(243-1)$$

$$= \frac{968}{5}$$

Evaluate.

$$\int\left(2x^2+1\right)^3 x\,dx \Rightarrow \frac{1}{4}\int u^3\,du$$

$$= \frac{1}{4}\left(\frac{1}{4}u^4\right)+C$$

$$= \frac{1}{16}u^4+C$$

$$\Rightarrow \frac{1}{16}\left(2x^2+1\right)^4+C$$

$u = 2x^2+1$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$$\cancel{D_x}\int_{2x}^{2x^3}\left(t^2+1\right)dt = \left(\tfrac{1}{3}t^3+t\right)\Big|_{2x}^{2x^3}$$

$$= \left[\tfrac{1}{3}(2x^3)^3+2x^3\right]-\left[\tfrac{1}{3}(2x)^3+2x\right]$$

$$= \tfrac{1}{3}(8x^9)+2x^3-\tfrac{1}{3}(8x^3)-2x$$

$$= \tfrac{8}{3}x^9+2x^3-\tfrac{8}{3}x^3-2x$$

$$= \tfrac{8}{3}x^9-\tfrac{2}{3}x^3-2x$$

$$D_x\left(\tfrac{8}{3}x^9-\tfrac{2}{3}x^3-2x\right) = 24x^8-2x^2-2$$

$$D_x\int_{2x}^{2x^3}\left(t^2+1\right)dx = \left[(2x^3)^2+1\right]\cdot 6x^2-\left[(2x)^2+1\right]\cdot 2$$

$$= (4x^6+1)6x^2-(4x^2+1)2$$

$$= 24x^8+6x^2-8x^2-2$$

$$= 24x^8-2x^2-2$$

$$\frac{d}{dt}\int_{2}^{6}\left(t^2+1\right)dt = (6^2+1)\cdot 0-(2^2+1)\cdot 0$$

$$= 0$$

$$\cancel{\frac{d}{dt}}\int \left(t^2+1\right)dt = \frac{1}{3}t^3+t+C$$

$$\frac{d}{dt}\left(\frac{1}{3}t^3+t+C\right) = t^2+1$$

Evaluate.

$$\frac{d}{dx}\int_4^{2x^3}\left(t^2+1\right)dt = \left[(2x^3)^2+1\right]\cdot 6x^2 - \left[(4)^2+1\right]\cdot 0$$

$$= (4x^6+1)6x^2$$

$$= 24x^8+6x^2$$

Find the average value of $f(x) = 2x^3$ on [1,3].

$$f(c) = \frac{1}{3-1}\int_1^3 2x^3\,dx$$

$$= \frac{1}{2}\left(\frac{1}{2}x^4\right)\Big|_1^3$$

$$= \frac{1}{4}\left(3^4 - 1^4\right)$$

$$= \frac{1}{4}(81-1)$$

$$f(c) = 20$$

Evaluate.

$$\int_{-2}^{3} x^2\left(x^3+7\right)^3 dx$$

$u = x^3 + 7$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$x = 3$

$u = 27 + 7$

$x = -2$

$u = -8 + 7$

$$\Rightarrow \frac{1}{3}\int_{-1}^{34} u^3\,du$$

$$= \frac{1}{3}\left(\frac{1}{4}u^4\right)\Big|_{-1}^{34}$$

$$= \frac{1}{12}\left(34^4 - (-1)^4\right)$$

$$= \frac{1}{12}(1336336 - 1)$$

$$= \frac{1336335}{12}$$

$$= \frac{445445}{4}$$

Practice.

$$\int \frac{1}{(4x+13)^2}dx = \int (4x+13)^{-2}\,dx$$

$u = 4x+13$

$du = 4dx$

$\frac{1}{4}du = dx$

$$\Rightarrow \frac{1}{4}\int u^{-2}\,du$$

$$= \frac{1}{4}(-u^{-1}) + C$$

$$= -\frac{1}{4}u^{-1} + C$$

$$\Rightarrow -\frac{1}{4(4x+13)} + C$$

Evaluate.

$$\int \left(4x^{-3} - 12x^{-4}\right)dx = -2x^{-2} + 4x^{-3} + C$$

Evaluate.

$$\int (x^4+5)^2\,dx = \int (x^8+10x^4+25)\,dx$$

$$= \frac{1}{9}x^9 + 2x^5 + 25x + C$$

Evaluate.

$$\int_0^5 \sqrt{25-x^2}\,x\,dx$$

$u = 25 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$x = 5$

$u = 25 - 25$

$x = 0$

$u = 25 - 0$

$$\Rightarrow -\frac{1}{2}\int_{25}^{0} u^{\frac{1}{2}}\,du$$

$$= -\frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_{25}^{0}$$

$$= -\frac{1}{3}\left(0^{\frac{3}{2}} - 25^{\frac{3}{2}}\right)$$

$$= -\frac{1}{3}\left(0 - \frac{1}{125}\right)$$

$$= \frac{125}{3}$$

Evaluate.

$$\int_1^4 x^{-4}\,dx = -\frac{1}{3}u^{-3}\Big|_1^4$$

$$= -\frac{1}{3}\left(4^{-3} - 1^{-3}\right)$$

$$= -\frac{1}{3}\left(\frac{1}{64} - 1\right)$$

$$= -\frac{1}{\cancel{3}_{\,1}}\left(-\frac{\cancel{63}^{\,21}}{64}\right)$$

$$= \frac{21}{64}$$

Evaluate.

$$\int_0^2 x^2\sqrt{x^3+1}\,dx \quad \Rightarrow \frac{1}{3}\int_1^9 u^{\frac{1}{2}}\,du$$

$u = x^3 + 1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$x = 2$

$u = 8 + 1$

$x = 0$

$u = 0 + 1$

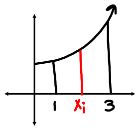
$$= \frac{1}{3}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_1^9$$

$$= \frac{2}{9}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$$

$$= \frac{2}{9}(27 - 1)$$

$$= \frac{52}{9}$$

Find the area under $f(x) = 9x^2 - 4x + 3$ on [1,3].



$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$x_0 = 1$$

$$x_1 = 1 + \Delta x = 1 + \frac{2}{n}$$

$$x_2 = 1 + 2\Delta x = 1 + \frac{4}{n}$$

$$\vdots$$

$$x_i = 1 + i\Delta x = 1 + \frac{2i}{n}$$

$$\vdots$$

$$x_n = 1 + n\Delta x = 1 + n\left(\frac{2}{n}\right) = 1 + 2 = 3 \checkmark$$

$$A_{R_i} = \Delta x \cdot f(x_i)$$

$$= \frac{2}{n}\left[9\left(1+\frac{2i}{n}\right)^2 - 4\left(1+\frac{2i}{n}\right) + 3\right]$$

$$= \frac{2}{n}\left[9\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right) - 4 - \frac{8i}{n} + 3\right]$$

$$= \frac{2}{n}\left[9 + \frac{36i}{n} + \frac{36i^2}{n^2} - 1 - \frac{8i}{n}\right]$$

$$= \frac{2}{n}\left(8 + \frac{28i}{n} + \frac{36i^2}{n^2}\right)$$

$$A_{R_i} = \frac{16}{n} + \frac{56i}{n^2} + \frac{72i^2}{n^3}$$

$$A = \sum_{i=1}^{n} A_{R_i} = \sum_{i=1}^{n}\left(\frac{16}{n} + \frac{56i}{n^2} + \frac{72i^2}{n^3}\right)$$

$$= \sum_{i=1}^{n}\frac{16}{n} + \sum_{i=1}^{n}\frac{56i}{n^2} + \sum_{i=1}^{n}\frac{72i^2}{n^3}$$

$$= n\left(\frac{16}{n}\right) + \frac{56}{n^2}\sum_{i=1}^{n} i + \frac{72}{n^3}\sum_{i=1}^{n} i^2$$

$$= 16 + \frac{\cancel{56}^{\,28}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}\;1}\right) + \frac{\cancel{72}^{\,12}}{n^{\cancel{3}\,2}}\left(\frac{\cancel{n}\overbrace{(n+1)(2n+1)}^{2n^2+3n+1}}{\cancel{6}\;1}\right)$$

$$= 16 + \frac{28}{n}(n+1) + \frac{12}{n^2}(2n^2+3n+1)$$

$$= 16 + 28 + \frac{28}{n} + 24 + \frac{36}{n} + \frac{12}{n^2}$$

$$= 68 + \frac{64}{n} + \frac{12}{n^2}$$

$$A = \lim_{n\to\infty}\left(68 + \frac{64}{n} + \frac{12}{n^2}\right)$$

$$= 68 + 0 + 0$$

$$A = 68u^2$$

Assignment:

Integration Review

#1-20

* TEST tomorrow *

H. Calculus ~ Integration Name____________________

Review

Evaluate each indefinite integral.

1) $\int (x^4-3)^{-5} \cdot 4x^3\,dx$

$-\frac{1}{4(x^4-3)^4}+C$

2) $\int \frac{8x}{(4x^2+1)^5}\,dx$

$-\frac{1}{4(4x^2+1)^4}+C$

3) $\int 8x\sqrt[3]{4x^2+5}\,dx$

$\frac{3}{4}(4x^2+5)^{\frac{4}{3}}+C$

4) $\int 25x^4(5x^5+3)^{-4}\,dx$

$-\frac{1}{3(5x^5+3)^3}+C$

5) $\int \frac{15x^2}{(5x^3+1)^4}\,dx$

$-\frac{1}{3(5x^3+1)^3}+C$

6) $\int 4x\sqrt[3]{2x^2-3}\,dx$

$\frac{3}{4}(2x^2-3)^{\frac{4}{3}}+C$

7) $\int 25x^4\sqrt{5x^5-1}\,dx$

$\frac{2}{3}(5x^5-1)^{\frac{3}{2}}+C$

8) $\int (x^3+4)^{-3}\cdot 3x^2\,dx$

$-\frac{1}{2(x^3+4)^2}+C$

9) $\int 16x^3(4x^4-5)^{\frac{6}{5}}\,dx$

$\frac{5}{11}(4x^4-5)^{\frac{11}{5}}+C$

10) $\int (2x^4+3)^5\cdot 8x^3\,dx$

$\frac{1}{6}(2x^4+3)^6+C$

Evaluate each definite integral.

11) $\int_{-7}^{-3} 2(x+2)^{\frac{1}{3}}\,dx$

$\frac{3-15\sqrt[3]{5}}{2}\approx -11.325$

12) $\int_{-6}^{-3} (-x^3-11x^2-35x-29)\,dx$

$-\frac{15}{4}=-3.75$

13) $\int_{-5}^{-3} \frac{1}{(2x+2)^2}\,dx$

$\frac{1}{16}\approx 0.063$

14) $\int_{0}^{1} -\frac{5}{(x+1)^3}\,dx$

$-\frac{15}{8}=-1.875$

15) $\int_{-4}^{-3} (-2x^2-12x-19)\,dx$

$-\frac{5}{3}\approx -1.667$

For each problem, find the average value of the function over the given interval.

16) $f(x)=-2x+2$; $[-2, 3]$

1

17) $f(x)=-2x$; $[-1, 3]$

-2

18) $f(x)=x-2$; $[4, 7]$

$\frac{7}{2}=3.5$

19) $f(x)=-2x+2$; $[-1, 4]$

-1

20) $f(x)=-x^2-2x-3$; $[-3, 1]$

$-\frac{10}{3}\approx -3.333$