

Today's Plan:

Learning Target (standard): I will integrate advanced and inverse trigonometric functions.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Trigonometric Integrals #1-16

$$1) -3\sec(5x^4 + 1) + C$$

$$2) -5\tan(x^4 - 5) + C$$

$$3) -5\sec(5x^5 + 1) + C$$

$$4) -2\tan(4x^3 + 1) + C$$

$$5) \cot(-3 + \ln(-5x)) + C$$

$$6) 4\sin(1 + \ln(-2x)) + C$$

$$7) -\cot(-2 + \ln(-3x)) + C$$

$$8) -2\tan(5 + \ln(-4x)) + C$$

$$9) -5\csc(\sin(-4x)) + C$$

$$10) -2\cot(\tan(-4x)) + C$$

$$11) -3\csc(\cot(-3x)) + C$$

$$12) -5\csc(\sec 5x) + C$$

$$13) -4\tan(e^{2x} - 3) + C$$

$$14) -5\tan(e^{2x} + 4) + C$$

$$15) 4\tan(e^{2x} + 2) + C$$

$$16) -5\csc(e^{5x} + 1) + C$$

Evaluate.

$$\int_0^2 x\sqrt{x^2+1} dx$$

$$u = x^2 + 1 \quad x=2 \quad u=4+1$$

$$du = 2x dx \quad x=0 \quad u=0+1$$

$$\frac{1}{2} du = x dx \quad u=0+1$$

$$\Rightarrow \frac{1}{2} \int_1^5 u^{\frac{1}{2}} du$$

$$= \frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \Big|_1^5$$

$$= \frac{1}{3} \left(5^{\frac{3}{2}} - 1^{\frac{3}{2}} \right)$$

$$= \frac{1}{3} (5\sqrt{5} - 1)$$

Evaluate.

$$\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx$$

$$= \int \frac{1}{2} dx - \frac{1}{2} \int \cos 2x dx$$

$$= \frac{1}{2} x \quad u = 2x \quad \Rightarrow -\frac{1}{4} \int \cos u du$$

$$du = 2 dx \quad = -\frac{1}{4} \sin u + C$$

$$\frac{1}{2} du = dx \quad \Rightarrow -\frac{1}{4} \sin 2x + C$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + C$$

Evaluate.

$$\int \cos^3 x dx \Rightarrow \int \cos x (\cos^2 x) dx$$

$$= \int \cos x (1 - \sin^2 x) dx$$

$$u = \sin x \quad \Rightarrow \int (1 - u^2) du$$

$$du = \cos x dx \quad = u - \frac{1}{3}u^3 + C$$

$$\Rightarrow \sin x - \frac{1}{3}\sin^3 x + C$$

Inverse Trig Derivatives:

$$\bullet y = \sin^{-1} f(x)$$

$$y' = \frac{f'(x)}{\sqrt{1 - f^2(x)}}$$

$$\bullet y = \cos^{-1} f(x)$$

$$y' = \frac{-f'(x)}{\sqrt{1 - f^2(x)}}$$

$$\bullet y = \tan^{-1} f(x)$$

$$y' = \frac{f'(x)}{1 + f^2(x)}$$

$$\bullet y = \sec^{-1} f(x)$$

$$y' = \frac{f'(x)}{|f(x)|\sqrt{f^2(x) - 1}}$$

Find the derivative.

$$y = \sin^{-1}(x^2) \quad y = \sin^{-1} f(x)$$

$$y' = \frac{2x}{\sqrt{1-(x^2)^2}}$$

$$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$$y' = \frac{2x}{\sqrt{1-x^4}}$$

$$\int \frac{2x}{\sqrt{1-x^4}} dx$$

~~$$u = 1-x^4$$

$$du = -4x^3 dx$$~~

$$= \int \frac{2x}{\sqrt{1-(x^2)^2}} dx$$

$$u = x^2$$

$$du = 2x dx$$

$$\Rightarrow \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1} u + C$$

$$\Rightarrow \sin^{-1}(x^2) + C$$

Find the derivative.

$$y = \tan^{-1} 5x$$

$$y' = \frac{5}{1+(5x)^2}$$

$$y' = \frac{5}{1+25x^2}$$

~~$$\int \frac{5}{1+25x^2} dx$$~~

~~$$u = 1+25x^2$$~~

~~$$du = 50x dx$$~~

Find the derivative.

$$y = \sec^{-1}(x^2 - x)$$

$$y' = \frac{2x-1}{|x^2-x|\sqrt{(x^2-x)^2-1}}$$

Inverse Trigonometric Integrals:

$$\bullet \int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + C$$

$$\bullet \int \frac{1}{1+u^2} du = \tan^{-1} u + C$$

$$\bullet \int \frac{1}{u\sqrt{u^2-1}} du = \sec^{-1} u + C$$

Evaluate.

$$\int \frac{x dx}{\sqrt{1-x^4}} \Rightarrow \int \frac{x}{\sqrt{1-(x^2)}} dx$$

$$\int \frac{x}{\sqrt{1-(x^2)}} dx \Rightarrow \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du$$

$$\frac{1}{2} du = x dx = \frac{1}{2} \sin^{-1} u + C$$

$$\Rightarrow \frac{1}{2} \sin^{-1}(x^2) + C$$

~~$u = 1 - x^4$~~
 ~~$du = -4x^3 dx$~~

$u = x^2$
 $du = 2x dx$

Evaluate.

$$\int \frac{e^x dx}{1+e^{2x}} = \int \frac{e^x}{1+(e^x)^2} dx$$

$$\int \frac{e^x}{1+(e^x)^2} dx \Rightarrow \int \frac{1}{1+u^2} du$$

$$= \tan^{-1} u + C$$

$$\Rightarrow \tan^{-1}(e^x) + C$$

~~$u = 1 + e^{2x}$~~
 ~~$du = 2e^{2x} dx$~~

$u = e^x$
 $du = e^x dx$

Evaluate.

$$\begin{aligned}\int \frac{3}{4x^2 + 4} dx &= \int \frac{3}{4(x^2+1)} dx \\ &= \frac{3}{4} \int \frac{1}{x^2+1} dx \\ &= \frac{3}{4} \tan^{-1} x + C\end{aligned}$$

Homework:

$$1) \int \frac{dx}{x\sqrt{x^2 - \pi}}$$

$$2) \int \frac{dx}{7 + x^2}$$

$$3) \int \frac{dx}{x(1 + \ln^2 x)}$$

$$4) \int \frac{\sec^2 x dx}{\sqrt{1 - \tan^2 x}}$$

$$5) \int \frac{dx}{\sqrt{9 - 4x^2}}$$

$$6) \int \frac{e^{3x} dx}{1 + e^{6x}}$$

Packet:

- p.341 #13
- p.350 #21,23a,35