Today's Plan:

Learning Target (standard): I will integrate advanced and inverse trigonometric functions.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

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7) $\frac{1}{2}\ln\left|\sec 2x+\tan 2x\right|+C$

8) $3\ln\left|\sin\sqrt[3]{x}\right|+C$

9) $-\frac{1}{2}\cot\left(x^2+1\right)+C$

10) $\frac{1}{2}x^2+\frac{1}{8}\ln\left|\csc 8x-\cot 8x\right|+C$

11) $\frac{1}{6}\sin 6x+C$

12) $\frac{1}{2}\ln\left|\sec 2x+\tan 2x\right|-\frac{1}{2}\sin 2x+C$

13) $\frac{1}{2}$

14) $-\frac{1}{2}\cot^2 x+C$

15) $\frac{1}{2}\ln\left|\sec 2x+\tan 2x\right|-\frac{1}{2}\sin 2x+C$

16) $-\frac{\sqrt{3}}{2}-\ln\left(2-\sqrt{3}\right)$

17) $-\frac{1}{8}$

18) $-\frac{1}{3}\cos^3 x+C$

19) $\ln\left|x+\cos x\right|+C$

20) $\ln\left|\sec e^x+\tan e^x\right|+C$

Evaluate.

$$\int e^x \cos\left(e^x\right) dx$$

$u = e^x$

$du = e^x dx$

$\Rightarrow \int \cos u \, du$

$= \sin u + C$

$\Rightarrow \sin e^x + C$

Evaluate.

$$\int (\sec x \tan x)\sqrt{4 + 3\sec x}\, dx$$

$u = 4 + 3\sec x$

$du = 3\sec x \tan x \, dx$

$\frac{1}{3} du = \sec x \tan x \, dx$

$\Rightarrow \frac{1}{3}\int u^{\frac{1}{2}} du$

$= \frac{1}{3}\left(\frac{2}{3} u^{\frac{3}{2}}\right) + C$

$= \frac{2}{9}\sqrt{(4 + 3\sec x)^3} + C$

Evaluate.

$$\int \tan^2 x \sec^2 x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$\Rightarrow \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$\Rightarrow \frac{1}{3}\tan^3 x + C$

Evaluate.

$$\int \frac{\sin 2x - \cos 2x}{\sin 2x + \cos 2x}\,dx$$

$u = \sin 2x + \cos 2x$

$du = (2\cos 2x - 2\sin 2x)\,dx$

$du = -2(\sin 2x - \cos 2x)\,dx$

$-\frac{1}{2}du = (\sin 2x - \cos 2x)\,dx$

$\Rightarrow -\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}\ln|u| + C$

$\Rightarrow -\frac{1}{2}\ln|\sin 2x + \cos 2x| + C$

Helpful Trig Identities:

- $\sin^2 x = \frac{1-\cos 2x}{2}$
- $\sin^2 x = 1-\cos^2 x$
- $\cos^2 x = \frac{1+\cos 2x}{2}$
- $\cos^2 x = 1-\sin^2 x$
- $\sin^2 x + \cos^2 x = 1$
- $\cot^2 x = \csc^2 x - 1$
- $\tan^2 x = \sec^2 x - 1$

Helpful Hints:

- If you have a single trig function raised to an even power that cannot be integrated as it is, rewrite the function in terms of another function that can be integrated

- If you have a single trig function raised to an odd power that cannot be integrated as it is, factor out one of the functions and rewrite the remaining in terms of another and use substitution

Evaluate.

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$$= \int \frac{1}{2}\,dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2}x$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\Rightarrow -\frac{1}{4}\int \cos u\,du$$

$$= -\frac{1}{4}\sin u + C$$

$$\Rightarrow -\frac{1}{4}\sin 2x + C$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

Evaluate.

$$\int \sin^3 x\,dx = \int \sin x \cdot \sin^2 x\,dx$$

$$= \int \sin x\,(1-\cos^2 x)\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$\Rightarrow -\int (1-u^2)\,du$$

$$= -\left(u - \frac{1}{3}u^3\right) + C$$

$$= -u + \frac{1}{3}u^3 + C$$

$$\Rightarrow -\cos x + \frac{1}{3}\cos^3 x + C$$

Homework:

1) $\int \sin^4 x dx$

2) $\int \cos^4 x dx$

3) $\int \cos^4 x \sin x dx$

4) $\int \sin^3 x \cos^5 x dx$