Today's Plan:

Learning Target (standard): I will solve real-world optimization application problems.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

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* be sure to draw and label diagrams when appropriate - this is how the variables are defined *

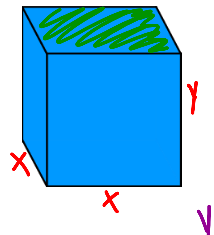
$1)1st = 20; 2nd = -20$

$2)1st = 20; 2nd = 20$

$3)l = 2\,ft; w = 2\,ft; h = 1\,ft$

$6)w = 2\sqrt{30}in; l = \frac{3\sqrt{30}}{2}in$

3) If a box with a square base and open top is to have a volume of 4 cubic feet, find the dimensions that require the least material.



$V = 4 ft^3$

$x^2 y = 4$

$\boxed{y = \frac{4}{x^2}}$

$SA = x^2 + 4xy$

$SA(x) = x^2 + 4x\left(\frac{4}{x^2}\right)$

$SA(x) = x^2 + 16x^{-1}$

$SA'(x) = 2x - 16x^{-2}$

$0 = 2x^{-2}(x^3 - 8)$

$0 = 2x^{-2}(x-2)(x^2+2x+4)$

Critical #s:

$x = \cancel{0}, 2$

$SA''(x) = 2 + 32x^{-3}$

$SA''(2) = 2 + 32(2)^{-3}$

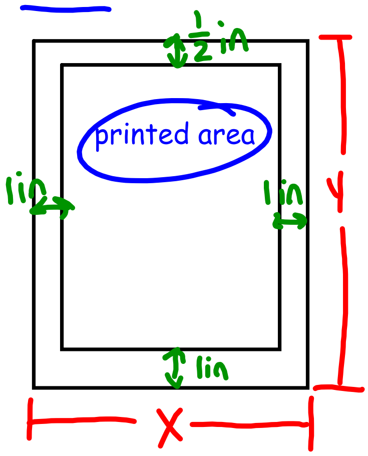
$SA''(2) > 0$

min @ $x = 2$

$y = \frac{4}{2^2} = 1$

∴ The dimensions that minimize the amount of material are $x = 2$ ft & $y = 1$ ft.

6) A page of a book is to have an area of 90 square inches with 1-inch margins at the bottom and sides and a $^1/_2$ - inch margin at the top. Find the dimensions of the page which will allow the largest printed area.



$A = xy$

$90 = xy$

$y = \frac{90}{x}$

$\boxed{y = 90x^{-1}}$

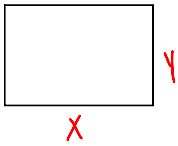
$AP = (x-2)\left(y - \frac{3}{2}\right)$

$AP(x) = (x-2)\left(90x^{-1} - \frac{3}{2}\right)$

$x = 2\sqrt{30}$ in

$y = \frac{3\sqrt{30}}{2}$ in

A couple has 3200 feet of fencing and wants to fence in a rectangular section of their backyard for their 7 dogs. What are the dimensions that will allow for the largest area?



$2x+2y=3200$

$2y=-2x+3200$

$\boxed{y=-x+1600}$

$A=xy$

$A(x)=x(-x+1600)$

$A(x)=-x^2+1600x$

$A'(x)=-2x+1600$

$0=-2x+1600$

$2x=1600$

$x=800$

Critical #:

$x=800$

$A''(x)=-2$

$A''(800)=-2<0$

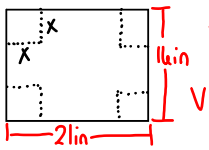
max @ $x=800$

$y=-800+1600$

$y=800$

$\therefore$ The dimensions that will maximize the area are $x=800$ ft and $y=800$ ft.

An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square that will produce the box with the largest volume.



$V=lwh$

$V(x)=(21-2x)(16-2x)x$

$V(x)=(21-2x)(16x-2x^2)$

$V(x)=4x^3-74x^2+336x$

$V'(x)=12x^2-148x+336$

$0=4(3x^2-37x+84)$

$0=4(3x-28)(x-3)$

Critical #s:

$x=\cancel{\frac{28}{3}},3$

$V''(x)=24x-148$

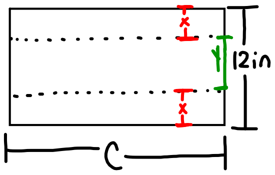
$V''(3)=24(3)-148$

$V''(3)=72-148<0$

max @ $x=3$

A long rectangular sheet of metal 12 inches wide is to be made into a rain gutter by turning up sides at right angles to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

$y = 12 - 2x$



$V = lwh$

*C - constant length every time

$V(x) = C(12-2x)x$

$V(x) = 12Cx - 2Cx^2$

$V'(x) = 12C - 4Cx$

$0 = 4C(3-x)$

Critical #s:

$x = 3$

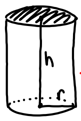
$V''(x) = -4C$

$V''(3) < 0$

max @ $x = 3$

$x = 3\text{in}$

A cylindrical container, open at the top with a capacity of 24π cubic inches, is to be manufactured. If the cost of the material used for the bottom of the container is three times that used for the curved part and if there is no waste of material, find the dimensions which will minimize the cost.



$V = 24\pi \text{ in}^3$

$V = \pi r^2 h$

$\boxed{24\pi = \pi r^2 h}$

material cost "C" positive

$h = \frac{24}{r^2}$

$\boxed{h = 24r^{-2}}$

$SA = \pi r^2 + 2\pi rh$

$C = 3C\pi r^2 + C(2\pi rh)$

$C(r) = 3C\pi r^2 + 2\pi Cr(24r^{-2})$

$C(r) = 3C\pi r^2 + 48\pi Cr^{-1}$

$C'(r) = 6C\pi r - 48\pi Cr^{-2}$

$0 = 6\pi Cr^{-2}(r^3 - 8)$

Critical #s: $r^3 - 8 = 0$, $r = 2$

$r = \cancel{0}, 2$ *cannot have a radius of 0in

$C''(r) = 6C\pi + 96\pi Cr^{-3}$

$C''(2) = 6C\pi + \frac{96\pi C}{8} > 0$ min @ $r = 2$

$h = \frac{24}{r^2} = \frac{24}{4} = 6$

$r = 2\text{in}$

$h = 6\text{in}$

Assignment:

Applications of Extrema

#1 & 2