Today's Plan:

Learning Target (standard): I will solve real-world related rate application problems.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Rates of Change Practice:

$$1a) V'(3) = -32\pi \frac{ft^3}{hr}$$

$$b) r'(3) = -2 \frac{ft}{hr}$$

$$c) SA'(3) = -32\pi \frac{ft^2}{hr}$$

4) hits the ground when:

$$s(t) = 0$$

$$t = 0, 25 \sec$$

velocity it hits the ground:

$$v(25) = -400 \frac{ft}{\sec}$$

maximum altitude when:

$$v(t) = 0$$

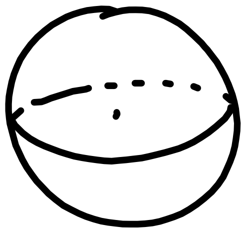
$$t = \frac{25}{2} \sec$$

maximum altitude is:

$$s\left(\frac{25}{2}\right) = 2500\, ft$$

$$a(t) = -32 \frac{ft}{\sec^2}$$

The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change of the surface area of the sphere?



$SA = 4\pi r^2$

$\frac{dr}{dt} = -2 \text{ cm/sec}$

$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dSA}{dt} = ?$ when $r = 3\text{cm}$

$\frac{dSA}{dt} = 8\pi(3)(-2)$

$\frac{dSA}{dt} = -48\pi \text{ cm}^2/\text{sec}$

Related Rates

- situations that are described in terms of how different quantities change with respect to each other

A 20 ft ladder leans against a vertical building. If the bottom slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the ladder top is 12 ft above the ground?

https://www.khanacademy.org/math/ap-calculus-ab/derivative-applications-ab/related-rates-ab/v/falling-ladder-related-rates

Systematic Procedure for Solving Related Rate Problems

1) Let t denote the elapsed time. Draw a diagram that is valid for all $t > 0$. Label those quantities whose values do not change as t increases with their given constant values.

2) State what is given about the variables and what information is wanted about them. This information will be in the form of derivatives with respect to t.

3) Write an equation relating variables that is valid at all times $t > 0$, not just at some particular instant.

4) Differentiate the equation found in the previous step implicitly with respect to time. The resulting equation, containing derivatives with respect to t, is true for all $t > 0$.

Systematic Procedure for Solving Related Rate Problems

5) Substitute in the equation found in step 4 all data that are valid *at the particular instant* for which the answer to the problem is required.

6) Solve for the desired derivative.

Related Rates

- situations that are described in terms of how different quantities change with respect to each other

A 20 ft ladder leans against a vertical building. If the bottom slides away from the building horizontally at a rate of 2 ft/sec how fast is the ladder sliding down the building when the ladder top is 12 ft above the ground?

$x^2+y^2=20^2$

$\frac{dy}{dt}=?$

$\frac{dx}{dt}=2\ \text{ft/sec}$

when $y=12$

$x^2+y^2=400$

$x^2+12^2=400$

$x^2+144=400$

$\sqrt{x^2}=\pm\sqrt{256}$

$x=16, \cancel{-16}$

$x^2+y^2=20^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

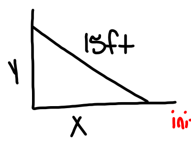
$2(16)(2)+2(12)\frac{dy}{dt}=0$

$64+24\frac{dy}{dt}=0$

$24\frac{dy}{dt}=-64$

$\frac{dy}{dt}=-\frac{8}{3}\ \text{ft/sec}$

A 15 foot ladder is resting against a wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of $^1/_4$ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?



initially $x=10$ ft

$\frac{dx}{dt}=-\frac{1}{4}\ \text{ft/sec}$

$x^2+y^2=15^2$

$\frac{dy}{dt}=?$

when $t=12$ sec

$x=12\left(-\frac{1}{4}\right)=-3$

$x=7$ ft

$x^2+y^2=15^2$

$49+y^2=225$

$\sqrt{y^2}=\pm\sqrt{176}$

$y=4\sqrt{11}, \cancel{-4\sqrt{11}}$

$x^2+y^2=225$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$2(7)\left(-\frac{1}{4}\right)+2(4\sqrt{11})\frac{dy}{dt}=0$

$-\frac{7}{2}+8\sqrt{11}\frac{dy}{dt}=0$

$8\sqrt{11}\frac{dy}{dt}=\frac{7}{2}$

$\frac{dy}{dt}=\frac{7}{2}\cdot\frac{1}{8\sqrt{11}}$

$\frac{dy}{dt}=\frac{7\sqrt{11}}{16\cdot 11}$

$\frac{dy}{dt}=\frac{7\sqrt{11}}{176}\ \text{ft/sec}$

Assignment:

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