Today's Plan:

Learning Target (standard): I will estimate the area under a curve using Riemann sums.

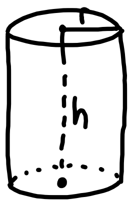
Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Water is draining from a cylindrical tank at 4 liters/second. If the radius of the tank is 2 centimeters, how fast is the surface dropping?



$\frac{dV}{dt} = -4\ L/sec$

$\boxed{1L = 1000cm^3}$

$r = 2cm$ $\frac{dh}{dt} = ?$

$\frac{dV}{dt} = -4000\ cm^3/sec$

$\frac{dr}{dt} = 0\ cm/sec$

*radius is constant

$V = \pi r^2 h$

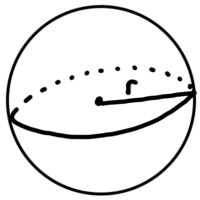
$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

$-4000 = 2\pi(2)(h)(0) + \pi(2)^2 \frac{dh}{dt}$

$-4000 = 4\pi \frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1000}{\pi}\ cm/sec$

A balloon is submerged in liquid nitrogen. The balloon's diameter contracts when it is cooled. The diameter of the sphere is decreasing at a rate of 4 cm/sec. How fast is the surface area changing when the radius is 10 cm?



$\frac{dd}{dt} = -4 \text{ cm/sec} \rightarrow \frac{dr}{dt} = -2 \text{ cm/sec}$

$\frac{dSA}{dt} = ?$ when $r = 10$ cm

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$

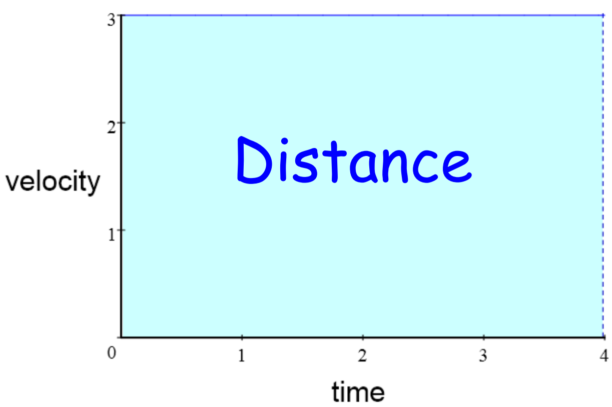
$\frac{dSA}{dt} = 8\pi(10)(-2)$

$\frac{dSA}{dt} = -160\pi \text{ cm}^2/\text{sec}$

Consider an object moving at a constant rate of 3 ft/sec.

Since rate · time = distance: $3t = d$

If we draw a graph of the velocity, the distance that the object travels is equal to the area under the line.



After 4 seconds, the object has gone 12 feet.

$3\frac{\text{ft}}{\text{sec}} \cdot 4 \text{ sec} = 12 \text{ ft}$

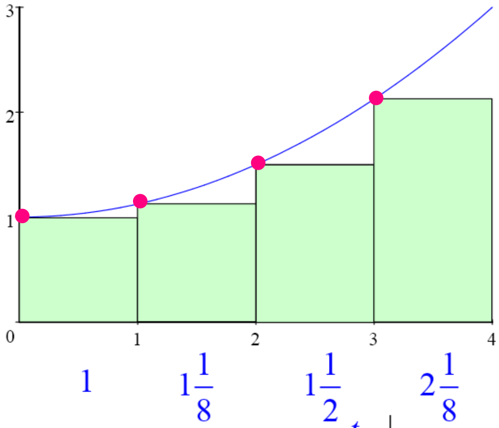
→

Riemann Sums:

If the velocity is not constant, we might guess that the distance traveled is still equal to the area under the curve.

(The units work out.)

Example: $V = \frac{1}{8}t^2 + 1$



t	v
0	1
1	$1\frac{1}{8}$
2	$1\frac{1}{2}$
3	$2\frac{1}{8}$

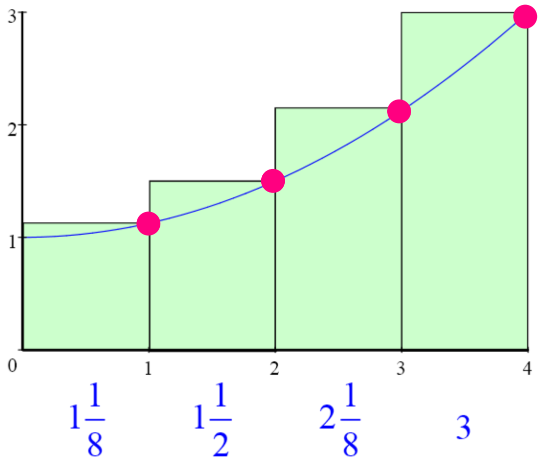
We could estimate the area under the curve by drawing rectangles touching at their left corners.

This is called the Left-hand Rectangular Approximation Method (LRAM).

Approximate area: $1 + 1\frac{1}{8} + 1\frac{1}{2} + 2\frac{1}{8} = 5\frac{3}{4} = 5.75$

→

$V = \frac{1}{8}t^2 + 1$



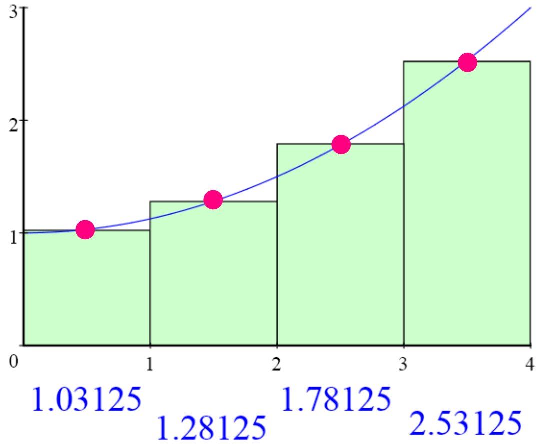
We could also use a Right-hand Rectangular Approximation Method (RRAM).

Approximate area: $1\frac{1}{8} + 1\frac{1}{2} + 2\frac{1}{8} + 3 = 7\frac{3}{4} = 7.75$

→

t	v
0.5	1.03125
1.5	1.28125
2.5	1.78125
3.5	2.53125

$$V = \frac{1}{8}t^2 + 1$$



Another approach would be to use rectangles that touch at the midpoint. This is the Midpoint Rectangular Approximation Method (MRAM).

Approximate area:
6.625

In this example there are four subintervals.
As the number of subintervals increases, so does the accuracy.

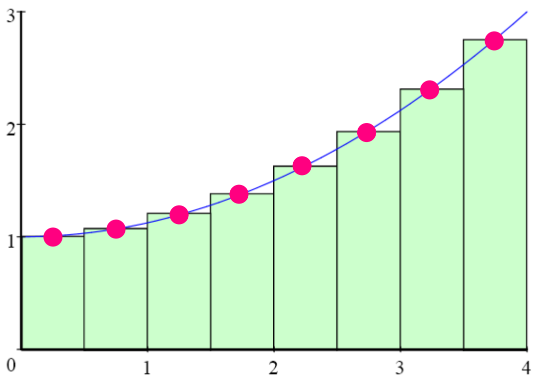
With 8 subintervals:

$$V = \frac{1}{8}t^2 + 1$$

t	v
0.25	1.00781
0.75	1.07031
1.25	1.19531
1.75	1.38281
2.25	1.63281
2.75	1.94531
3.25	2.32031
3.75	2.75781

$$13.31248 \times 0.5 = 6.65624$$

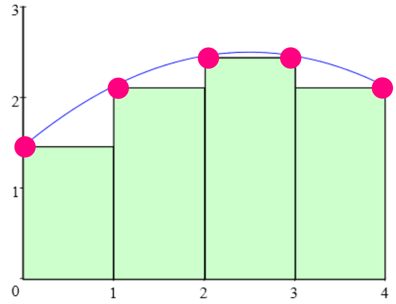
width of subinterval



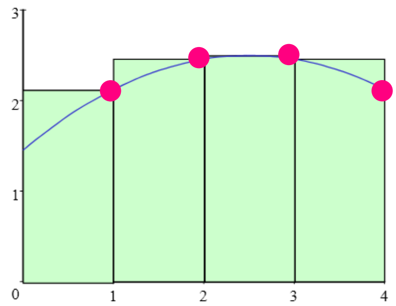
Approximate area:
6.65624

The exact answer for this problem is $6.\overline{6}$.

Inscribed rectangles are all below the curve:



Circumscribed rectangles are all above the curve:

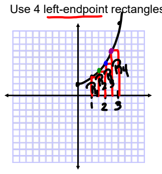


$$A_{trap} = \frac{1}{2}(b_1 + b_2)h$$

→

Estimate the area under $y = x^2 + 2$ on the interval [1,3].

Use 4 left-endpoint rectangles.



* intervals must be equal widths *

$$\Delta x = \frac{b-a}{n} \quad [a,b]$$

# of intervals

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$A_{R_1} = \frac{1}{2}f(1) = \frac{1}{2}(1^2+2) = \frac{1}{2}(3)$$

$$A_{R_1} = \frac{3}{2}u^2$$

$$A_{R_2} = \frac{1}{2}f\left(\frac{3}{2}\right) = \frac{1}{2}\left(\left(\frac{3}{2}\right)^2+2\right) = \frac{1}{2}\left(\frac{9}{4}+\frac{8}{4}\right)$$

$$A_{R_2} = \frac{17}{8}u^2$$

$$A_{R_3} = \frac{1}{2}f(2) = \frac{1}{2}(2^2+2) = \frac{1}{2}(6)$$

$$A_{R_3} = 3u^2$$

$$A_{R_4} = \frac{1}{2}f\left(\frac{5}{2}\right) = \frac{1}{2}\left(\left(\frac{5}{2}\right)^2+2\right) = \frac{1}{2}\left(\frac{25}{4}+\frac{8}{4}\right)$$

$$A_{R_4} = \frac{33}{8}u^2$$

$$A \approx A_{R_1} + A_{R_2} + A_{R_3} + A_{R_4}$$

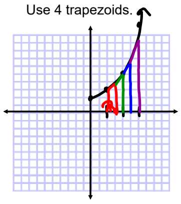
$$\approx \frac{3}{2} + \frac{17}{8} + 3 + \frac{33}{8}$$

$$\approx \frac{12+17+24+33}{8}$$

$$A \approx \frac{43}{4}u^2$$

Estimate the area under $y = x^2 + 2$ on the interval [1,3].

Use 4 trapezoids.



$\Delta x = \frac{3-1}{4} = \frac{1}{2} = h$

$A_{R_1} = \frac{1}{2}\left(f(1)+f(\tfrac{3}{2})\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(3+\frac{17}{4}\right)$

$= \frac{1}{4}\left(\frac{29}{4}\right)$

$A_{R_1} = \frac{29}{16}u^2$

$A_{R_2} = \frac{1}{2}\left(f(\tfrac{3}{2})+f(2)\right)\frac{1}{2}$

$= \frac{1}{4}\left(\frac{17}{4}+6\right)$

$= \frac{1}{4}\left(\frac{41}{4}\right)$

$A_{R_2} = \frac{41}{16}u^2$

$A_{R_3} = \frac{1}{2}\left(f(2)+f(\tfrac{5}{2})\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(6+\frac{33}{4}\right)$

$= \frac{1}{4}\left(\frac{57}{4}\right)$

$A_{R_3} = \frac{57}{16}u^2$

$A_{R_4} = \frac{1}{2}\left(f(\tfrac{5}{2})+f(3)\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(\frac{33}{4}+11\right)$

$= \frac{1}{4}\left(\frac{77}{4}\right)$

$A_{R_4} = \frac{77}{16}u^2$

$A \approx A_{R_1}+A_{R_2}+A_{R_3}+A_{R_4}$

$\approx \frac{29}{16}+\frac{41}{16}+\frac{57}{16}+\frac{77}{16}$

$A \approx \frac{204}{16}$

$A \approx \frac{51}{4}u^2$

Homework:

1. Estimate the area under y = x^3 on the interval [2,3] using:

   a. 4 inscribed rectangles
   b. 4 circumscribed rectangles
   c. 4 trapezoids
   d. 4 mid-points

2. Estimate the area under $y = 2x - x^2$ on the interval [1,2] using:

   a. 4 inscribed rectangles
   b. 4 circumscribed rectangles
   c. 4 trapezoids

3. Describe the factors that play a role in estimating the area under a curve using Riemann sums. Provide an example.