Today's Plan:

Learning Target (standard): I will find the area under a curve using Riemann sums.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

p.214 #1-10

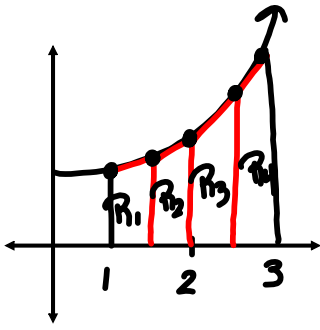|	
1) -5	6) 10
2) 12	7) 510
3) 34	8) $^{49}/_4$
4) 10	9) 500
5) 40	10) 2000

Estimate the area under $y = x^2 + 2$ on the interval $[1,3]$.

Use 4 trapezoids.

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$A_{R_1} = \frac{1}{2}\left(f(1) + f\left(\frac{3}{2}\right)\right)\frac{1}{2}$$

$$= \frac{1}{4}\left(3 + \frac{9}{4} + 2\right)$$

$$= \frac{1}{4}\left(\frac{20}{4} + \frac{9}{4}\right)$$

$$A_{R_1} = \frac{29}{16} \, u^2$$

$$A_{R_2} = \frac{1}{2}\left(f\left(\frac{3}{2}\right) + f(2)\right)\frac{1}{2}$$

$$= \frac{1}{4}\left(\frac{9}{4} + 2 + 6\right)$$

$$= \frac{1}{4}\left(\frac{9}{4} + \frac{32}{4}\right)$$

$$A_{R_2} = \frac{41}{16} \, u^2$$

$$A_{R_3} = \frac{1}{2}\left(f(2) + f\left(\frac{5}{2}\right)\right)\frac{1}{2}$$

$$= \frac{1}{4}\left(6 + \frac{25}{4} + 2\right)$$

$$= \frac{1}{4}\left(\frac{32}{4} + \frac{25}{4}\right)$$

$$A_{R_3} = \frac{57}{16} \, u^2$$

$$A_{R_4} = \frac{1}{2}\left(f\left(\frac{5}{2}\right) + f(3)\right)\frac{1}{2}$$

$$= \frac{1}{4}\left(\frac{25}{4} + 2 + 11\right)$$

$$= \frac{1}{4}\left(\frac{25}{4} + \frac{52}{4}\right)$$

$$A_{R_4} = \frac{77}{16} \, u^2$$

$$A \approx \frac{29}{16} + \frac{41}{16} + \frac{57}{16} + \frac{77}{16}$$

$$\approx \frac{204}{16}$$

$$A \approx \frac{51}{4} \, u^2$$

Find the sum (use the formula):

$$\sum_{i=1}^{20} i^3 = \left[\frac{20(20+1)}{2} \right]^2$$

$$= \left(\frac{20(21)}{2} \right)^2$$

$$= \left(10(21) \right)^2$$

$$= (210)^2$$

$$= 44100$$

Find the sum:

$$\sum_{i=1}^{n}\left(\frac{25}{n}\right) = \frac{25}{n} + \frac{25}{n} + \frac{25}{n} + \cdots + \frac{25}{n}$$

$$= n\left(\frac{25}{n}\right)$$

$$= 25$$

Find the sum (use the formula):

$$\sum_{i=1}^{10} i^2 = \frac{10(10+1)(20+1)}{6}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{\overset{5}{\cancel{10}} \cdot 11 \cdot \cancel{21}^{7}}{\cancel{6}\cancel{21}}$$

$$= 385$$

Find the sum (use the formula):

$$\sum_{i=1}^{6} i^3 = \left[\frac{6\,(6+1)}{2}\right]^2$$

$$= \left[\frac{6(7)}{21}\right]^2$$

$$= 21^2$$

$$= 441$$

Find the sum (use the formula):

$$\sum_{i=1}^{50} i^2 = \frac{50(50+1)(100+1)}{6}$$

$$= \frac{\overset{25}{\cancel{50}}(\overset{17}{\cancel{51}})(101)}{\cancel{6}\;\cancel{3}\;1}$$

$$= 42{,}925$$

Find/calculate the area under $y = x^2 + 2$ on the interval [1,3].
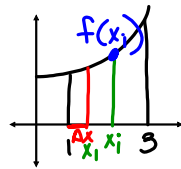
$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$* \, n = $ infinite # of subintervals

$$x_0 = 1 = a$$
$$x_1 = 1 + \Delta x = 1 + \frac{2}{n}$$
$$x_2 = 1 + 2\Delta x = 1 + \frac{4}{n}$$

$$\boxed{x_i = 1 + i\Delta x = \boxed{1 + \frac{2i}{n}}}$$

$$x_n = 1 + n\Delta x = 1 + n\left(\frac{2}{n}\right)$$
$$= 1 + 2$$
$$= 3 = b$$

$$A_{R_i} = \Delta x \cdot f(x_i)$$
$$= \frac{2}{n}\left[\left(1 + \frac{2i}{n}\right)^2 + 2\right]$$
$$= \frac{2}{n}\left[1 + \frac{4i}{n} + \frac{4i^2}{n^2} + 2\right]$$
$$= \frac{2}{n}\left[3 + \frac{4i}{n} + \frac{4i^2}{n^2}\right]$$

$$A_{R_i} = \frac{6}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}$$

$$A = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n}\left(\frac{6}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)$$

$$= \sum_{i=1}^{n}\frac{6}{n} + \sum_{i=1}^{n}\frac{8i}{n^2} + \sum_{i=1}^{n}\frac{8i^2}{n^3}$$

$$= 6 + \frac{8}{n^2}\sum_{i=1}^{n}i + \frac{8}{n^3}\sum_{i=1}^{n}i^2$$

$$= 6 + \frac{8^4}{n^2}\left(\frac{n(n+1)}{2_1}\right) + \frac{8^4}{n^3}\left(\frac{n(n+1)(2n+1)}{6_3}\right)$$

$$= 6 + \frac{4}{n}(n+1) + \frac{4}{3n^2}\left(2n^2 + 3n + 1\right)$$

$$= 6 + 4 + \frac{4}{n} + \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$$
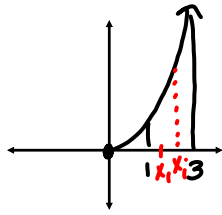
$$= \frac{38}{3} + \frac{8}{n} + \frac{4}{3n^2}$$

$$A = \lim_{n \to \infty}\left(\frac{38}{3} + \frac{8}{n} + \frac{4}{3n^2}\right)$$

$$= \frac{38}{3} + 0 + 0$$

$$\boxed{A = \frac{38}{3} \, u^2}$$

Find the area under $y = 3x^2 + 2x$ on the interval [1,3].

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}$$

$$X_1 = 1$$

$$X_1 = 1 + \Delta x = 1 + \frac{2}{n}$$

$$X_2 = 1 + 2\Delta x = 1 + \frac{4}{n}$$

$$X_i = 1 + i\Delta x = 1 + \frac{2i}{n}$$

$$X_n = 1 + n\Delta x = 1 + 2 = 3 \checkmark$$

$$A_{R_i} = \Delta x \cdot f(x_i)$$

$$= \frac{2}{n}\left[3\left(1+\frac{2i}{n}\right)^2 + 2\left(1+\frac{2i}{n}\right)\right]$$

$$= \frac{2}{n}\left[3\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right) + 2 + \frac{4i}{n}\right]$$

$$= \frac{2}{n}\left(3 + \frac{12i}{n} + \frac{12i^2}{n^2} + 2 + \frac{4i}{n}\right)$$

$$= \frac{2}{n}\left(5 + \frac{16i}{n} + \frac{12i^2}{n^2}\right)$$

$$A_{R_i} = \frac{10}{n} + \frac{32i}{n^2} + \frac{24i^2}{n^3}$$

$$A = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n}\left(\frac{10}{n} + \frac{32i}{n^2} + \frac{24i^2}{n^3}\right)$$

$$= \sum_{i=1}^{n}\frac{10}{n} + \sum_{i=1}^{n}\frac{32i}{n^2} + \sum_{i=1}^{n}\frac{24i^2}{n^3}$$

$$= 10 + \frac{32}{n^2}\sum_{i=1}^{n}i + \frac{24}{n^3}\sum_{i=1}^{n}i^2$$

$$= 10 + \frac{32^{16}}{n^{2}}\left(\frac{n(n+1)}{2_1}\right) + \frac{24^4}{n^{32}}\left(\frac{n(n+1)(2n+1)}{6_1}\right)$$

$$= 10 + \frac{16}{n}(n+1) + \frac{4}{n^2}(2n^2+3n+1)$$

$$= 10 + 16 + \frac{16}{n} + 8 + \frac{12}{n} + \frac{4}{n^2}$$

$$= 34 + \frac{28}{n} + \frac{4}{n^2}$$

$$A = \lim_{n\to\infty}\left(34 + \frac{28}{n} + \frac{4}{n^2}\right)$$

$$= 34 + 0 + 0$$

$$\boxed{A = 34 \, u^2}$$

Assignment:

p.214 #17-21