

Today's Plan:

Learning Target (standard): I will review all properties of finding derivatives.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

If the position function of a particle is given by $s(t) = 8t^2 - 2t$ feet, what is the velocity at a time of a seconds? When will the particle hit the ground? What velocity will it hit the ground? When will it reach its maximum height? When will the particle move in the positive direction? The negative direction?

$$\begin{aligned} \textcircled{1} v(t) &= s'(t) \\ v(t) &= 16t - 2 \\ v(a) &= 16a - 2 \text{ ft/sec} \end{aligned}$$

$$\begin{aligned} \textcircled{3} v\left(\frac{1}{4}\right) &= 16\left(\frac{1}{4}\right) - 2 \\ v\left(\frac{1}{4}\right) &= 4 - 2 \\ v\left(\frac{1}{4}\right) &= 2 \text{ ft/sec} \end{aligned}$$

hit the ground @ $\frac{1}{4}$ sec
with a velocity of 2 ft/sec

$$\begin{aligned} \textcircled{5} \text{ positive direction} \\ v(a) &> 0 \\ 16a - 2 &> 0 \\ a &> \frac{1}{8} \text{ sec} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \text{ hit ground when } s(t) &= 0 \\ 8t^2 - 2t &= 0 \\ 2t(4t - 1) &= 0 \\ t &= 0, \frac{1}{4} \\ t &= \frac{1}{4} \text{ sec} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \text{ maximum height} \\ v(a) &= 0 \\ 16a - 2 &= 0 \\ 16a &= 2 \\ a &= \frac{1}{8} \text{ sec} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \text{ negative direction} \\ v(a) &< 0 \\ 16a - 2 &< 0 \\ a &< \frac{1}{8} \text{ sec} \end{aligned}$$

Find the derivative using the definition and state 2 interpretations:

$$f(x) = \sqrt{3x+1}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x+3h+1} - \sqrt{3x+1}}{h} \cdot \frac{\sqrt{3x+3h+1} + \sqrt{3x+1}}{\sqrt{3x+3h+1} + \sqrt{3x+1}}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{3x+3h+1} - \cancel{3x+1}}{h(\sqrt{3x+3h+1} + \sqrt{3x+1})}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3x+3h+1} + \sqrt{3x+1})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h+1} + \sqrt{3x+1}}$$

$$= \frac{3}{\sqrt{3x+0+1} + \sqrt{3x+1}}$$

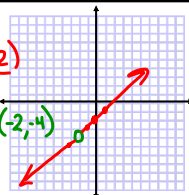
$$f'(x) = \frac{3}{2\sqrt{3x+1}}$$

∴ Since $f(x) = \sqrt{3x+1}$ and $f'(x) = \frac{3}{2\sqrt{3x+1}}$, the slope of the tangent line thru the point $(a, f(a))$ to $f(x) = \sqrt{3x+1}$ is $\frac{3}{2\sqrt{3a+1}}$. The velocity of a particle moving along $f(x) = \sqrt{3x+1}$ @ a time of "a" is $\frac{3}{2\sqrt{3a+1}}$.

Discuss the continuity:

$$f(x) = \frac{x^2 - 4}{x + 2} = \frac{(x+2)(x-2)}{x+2}$$

$$f(x) = x - 2$$



$$1) D: \{x \mid x \neq -2\}$$

$$2) \text{Continuous } \therefore (-\infty, -2) \cup (-2, \infty)$$

$$3) \text{Removable Discontinuity @ } x = -2.$$

$$a. f(-2) = \text{undefined}$$

$$b. \lim_{x \rightarrow -2^-} f(x) = -4$$

$$\lim_{x \rightarrow -2^+} f(x) = -4$$

$$\therefore \lim_{x \rightarrow -2} f(x) = -4$$

$$c. f(-2) \neq \lim_{x \rightarrow -2} f(x)$$

4) ∴ Since $f(-2)$ is undefined and the left-hand and right-hand limits exist and are equal but not infinite, there must be a removable discontinuity @ $x = -2$.

Evaluate the limit.

$$\begin{aligned}\lim_{x \rightarrow 3} \left(\frac{x^3 - 27}{x - 3} \right) &= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{x-3}} \\ &= \lim_{x \rightarrow 3} (x^2 + 3x + 9) \\ &= 3^2 + 3(3) + 9 \\ &= 27\end{aligned}$$

Evaluate the limit.

$$\begin{aligned}\lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x^2 - 4} \right) &= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{(x+2)\cancel{(x-2)}} \\ &= \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x+2} \\ &= \frac{4+4+4}{2+2} \\ &= 3\end{aligned}$$

Evaluate the limit.

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2} \cdot \frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2+3-4} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(\sqrt{x^2+3}+2)}{(x+1)\cancel{(x-1)}} \\ &= \lim_{x \rightarrow 1} \frac{\sqrt{x^2+3}+2}{x+1} \\ &= \frac{\sqrt{1+3}+2}{1+1} \\ &= \frac{2+2}{2} \\ &= 2 \end{aligned}$$

Evaluate the limit:

$$\lim_{x \rightarrow 2^+} (1 + \sqrt{x-2}) = 1 \quad \text{D: } \{x \mid x \geq 2\}$$

$$\lim_{x \rightarrow 2^-} (1 + \sqrt{x-2}) = \text{DNE}$$

$$\lim_{x \rightarrow 2} (1 + \sqrt{x-2}) = \text{DNE}$$

Evaluate:

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

$$\frac{1}{-\frac{1}{100}} = 1 \cdot -\frac{100}{1} = -100$$

$$\frac{1}{-\frac{1}{10000}} = 1 \cdot -\frac{10000}{1} = -10000$$

Evaluate:

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 4x + 5}}{x^2}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{5}{x^2}}}{\frac{x^2}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{1}{x^2} + \frac{4}{x} + \frac{5}{x^2}}}{1} = \frac{0}{1} = 0$$

$$\sqrt{x^4} = x^2$$

$$\sqrt{x^2} = x$$

$$\sqrt{x^6} = x^3$$

Evaluate:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 7x} - x) \cdot \frac{\sqrt{x^2 + 7x} + x}{\sqrt{x^2 + 7x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 7x - x^2}{\sqrt{x^2 + 7x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{7x}{\sqrt{x^2 + 7x} + x}$$

$$\sqrt{x^2} = x$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{7x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{7x}{x^2}} + \frac{x}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{7}{\sqrt{1 + \frac{7}{x}} + 1} = \frac{7}{1+1} = \frac{7}{2}$$

Find the derivative:

$$f(x) = 3x^2(x^3 + 1)^7$$

$$f'(x) = 6x(x^3 + 1)^7 + 7(x^3 + 1)^6(3x^2)(3x^2)$$

$$= 6x(x^3 + 1)^7 + 63x^4(x^3 + 1)^6$$

$$= 3x(x^3 + 1)^6 [2(x^3 + 1) + 21x^3]$$

$$= 3x(x^3 + 1)^6 (2x^3 + 2 + 21x^3)$$

$$f'(x) = 3x(x^3 + 1)^6 (23x^3 + 2)$$

Find the derivative:

$$f(x) = (3x^2) \left(x^{\frac{1}{2}} \right)$$

$$f(x) = 3x^{\frac{3}{2}}$$

$$f'(x) = \frac{15}{2} x^{\frac{1}{2}}$$

Find the derivative:

$$f(x) = (x+1)^x$$

$$\ln y = \ln(x+1)^x$$

$$\ln y = x \cdot \ln(x+1)$$

$$\frac{1}{y} y' = \frac{\ln(x+1)}{x+1} + \frac{x}{x+1}$$

$$\frac{1}{y} y' = \frac{x \ln(x+1) + \ln(x+1) + x}{x+1}$$

$$y' = \left[\frac{x \ln(x+1) + \ln(x+1) + x}{x+1} \right] y$$

$$y' = [x \ln(x+1) + \ln(x+1) + x] (x+1)^{x-1}$$

$$(x+1)^x \cdot \frac{(x+1)^x}{x+1}$$

$$\frac{m^x}{m^1} = m^{x-1}$$

Find the derivative:

$$f(x) = x^3 \sqrt[5]{2-x}$$

$$f(x) = x^3 (2-x)^{\frac{1}{5}}$$

$$f'(x) = 3x^2 (2-x)^{\frac{1}{5}} + \frac{1}{5} (2-x)^{-\frac{4}{5}} (-1) (x^3)$$

$$= 3x^2 (2-x)^{\frac{1}{5}} - \frac{1}{5} x^3 (2-x)^{-\frac{4}{5}}$$

$$= \frac{1}{5} x^2 (2-x)^{-\frac{4}{5}} [15(2-x) - x]$$

$$= \frac{1}{5} x^2 (2-x)^{-\frac{4}{5}} (30 - 15x - x)$$

$$f'(x) = \frac{x^2 (30 - 16x)}{5 \sqrt[5]{(2-x)^4}}$$