

Today's Plan:

Learning Target (standard): I will use Pascal's triangle to expand binomials.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Series Practice #1-10

1)168

2)310

3)99

4)3905

5)158

6)509

7)15

8)609

9)49

10)70

Find the sum of the series.

$$\begin{aligned}\sum_{k=2}^7 (2k - 4) &= [2(2) - 4] + [2(3) - 4] + [2(4) - 4] + \\ &\quad [2(5) - 4] + [2(6) - 4] + [2(7) - 4] \\ &= 0 + 2 + 4 + 6 + 8 + 10 \\ &= 30\end{aligned}$$

Find the sum of the series.

$$\begin{aligned}\sum_{k=4}^8 (3 - k) &= [3 - 4] + [3 - 5] + [3 - 6] + [3 - 7] \\ &\quad + [3 - 8] \\ &= -1 - 2 - 3 - 4 - 5 \\ &= -15\end{aligned}$$

Expand:

$$\begin{aligned}
 (x+1)^{\textcircled{3}} &= \underbrace{(x+1)(x+1)(x+1)}_{\text{row \#}} \\
 &= (x^2+x+x+1)(x+1) \\
 &= \underline{(x^2+2x+1)}(x+1) \\
 &= \underline{x^3} + \underline{x^2} + \underline{2x^2} + \underline{2x} + \underline{x} + 1 \\
 &= x^3 + 3x^2 + 3x + 1
 \end{aligned}$$

Maths at Home

Binomial Expansions

$(a+b)^{\textcircled{0}} = a+b$

$(a+b)^{\textcircled{2}} = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^{\textcircled{5}} = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

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The values in the nth row of Pascal's triangle (starting from 0) represent the coefficients in the expansion of $(a+b)^n$.

Exponent	Pascal's Triangle	Binomial Expansion
0	1	$(a+b)^0 = 1$
1	1 1	$(a+b)^1 = 1a + 1b$
2	1 2 1	$(a+b)^2 = 1a^2 + 2ab + 1b^2$
3	1 3 3 1	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$
4	1 4 6 4 1	$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$
5	1 5 10 10 5 1	$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

via OnlineMathLearning

Binomial Expansion Theorem:

$$(ax + by)^n = \binom{\#n - row}{1^{st} spot} (ax)^n (by)^0 + \binom{\#n - row}{2^{nd} spot} (ax)^{n-1} (by)^1 +$$

$$\binom{\#n - row}{3^{rd} spot} (ax)^{n-2} (by)^2 + \dots + \binom{\#n - row}{2^{nd} to - last} (ax)^1 (by)^{n-1} +$$

$$\binom{\#n - row}{last - spot} (ax)^0 (by)^n$$

$$(ax - by)^n = (ax + (-by))^n$$

Expand using the Binomial Theorem:

$$(x+1)^{\textcircled{3}} = \binom{\textcircled{3}}{\textcircled{1}} (x)^{\textcircled{3}} \binom{\textcircled{3}}{\textcircled{0}} + \binom{\textcircled{3}}{\textcircled{2}} (x)^{\textcircled{2}} \binom{\textcircled{3}}{\textcircled{1}} + \binom{\textcircled{3}}{\textcircled{1}} (x)^{\textcircled{1}} \binom{\textcircled{3}}{\textcircled{2}} + \binom{\textcircled{3}}{\textcircled{0}} (x)^{\textcircled{0}} \binom{\textcircled{3}}{\textcircled{3}}$$

$$= x^3 + 3x^2 + 3x + 1$$

(Note: The handwritten work includes a Pascal's triangle for row 3: 1, 3, 3, 1, with the 3s crossed out. The binomial coefficients in the expansion are underlined in green, and the powers of x and 1 are underlined in blue. The row number 3 is circled in red and labeled 'row#'. The binomial coefficients are also circled in red.)

Expand using the Binomial Theorem:

$$\begin{aligned}
 (x-2y)^4 &= (1)(x)^4(-2y)^0 + (4)(x)^3(-2y)^1 + (6)(x)^2(-2y)^2 \\
 &\quad + (4)(x)^1(-2y)^3 + (1)(x)^0(-2y)^4 \\
 &= x^4 + 4(x^3)(-2y) + 6(x^2)(4y^2) \\
 &\quad + 4(x)(-8y^3) + (1)(1)(16y^4) \\
 &= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4
 \end{aligned}$$

Binomial coefficients for $(x-2y)^4$ (row 4): 1, 4, 6, 4, 1. The coefficients 1, 4, 6, 4, 1 are circled in red in the original image.

Expand using the Binomial Theorem:

$$\begin{aligned}
 (2x-3)^5 &= (1)(2x)^5(-3)^0 + (5)(2x)^4(-3)^1 + (10)(2x)^3(-3)^2 \\
 &\quad + (10)(2x)^2(-3)^3 + (5)(2x)^1(-3)^4 + (1)(2x)^0(-3)^5 \\
 &= (1)(32x^5)(1) + (5)(16x^4)(-3) + (10)(8x^3)(9) \\
 &\quad + (10)(4x^2)(-27) + (5)(2x)(81) + (1)(1)(-243) \\
 &= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243
 \end{aligned}$$

Binomial coefficients for $(2x-3)^5$ (row 5): 1, 5, 10, 10, 5, 1. The coefficients 1, 5, 10, 10, 5, 1 are circled in red in the original image.

Expand using the Binomial Theorem:

$$\begin{aligned}
 (1 - 2y^3)^5 &= (1)(1)^5(-2y^3)^0 + (5)(1)^4(-2y^3)^1 + (10)(1)^3(-2y^3)^2 \\
 &\quad + (10)(1)^2(-2y^3)^3 + (5)(1)^1(-2y^3)^4 + (1)(1)^0(-2y^3)^5 \\
 &= (1)(1)(1) + (5)(1)(-2y^3) + (10)(1)(4y^6) + (10)(1)(-8y^9) \\
 &\quad + (5)(1)(16y^{12}) + (1)(1)(-32y^{15}) \\
 &= 1 - 10y^3 + 40y^6 - 80y^9 + 80y^{12} - 32y^{15}
 \end{aligned}$$

Assignment:

Binomial Expansion

#1-10

* Pick ANY 5 *