Today's Plan:

Learning Target (standard): I will discuss the continuity of a function.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Evaluate.

$$\lim_{x\to-\infty}\left(\frac{2^{-x}}{2^{x}}\right) = \lim_{x\to-\infty}\frac{1}{2^{x}\cdot 2^{x}}$$

$$= \lim_{x\to-\infty}\frac{1}{2^{2x}} \qquad \to \lim_{x\to\infty}\frac{1}{2^{2x}}$$

$$= \frac{1}{2^{-\infty}} \qquad = \frac{1}{2^{\infty}}$$

$$= 2^{\infty} \qquad = 0$$

$$= \infty$$

Evaluate.

$$\lim_{x\to 4^-}\left(\frac{\sqrt{16-x^2}}{4-x}\right) = \infty$$

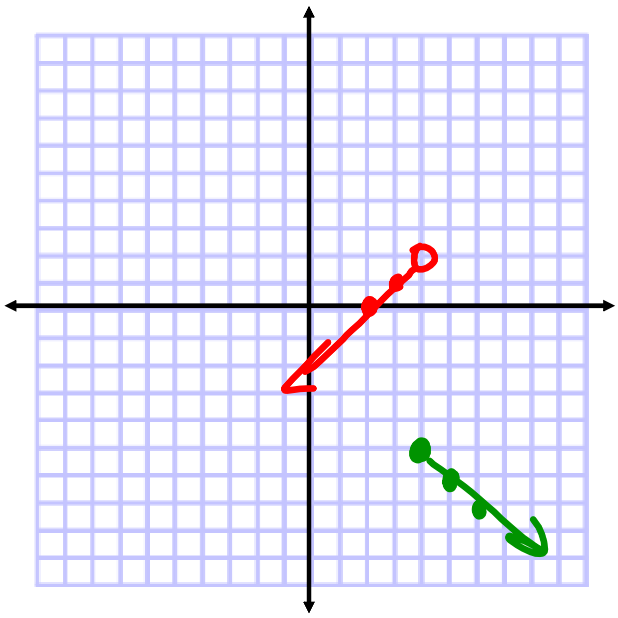
VA: $x=4$

Graph. Find domain and range.

$$f(x)=\begin{cases} x-2, & \boxed{x<4} \\ -x-1, & \boxed{x\geq 4} \end{cases}$$

D: $\mathbb{R}$

R: $\{y \mid y<2\}$



Definition of Continuity

- A function f(x) is continuous at x = ***a*** if:

  1) ***a*** is in the domain of f(x)

  2) limit as x approaches ***a*** of f(x) exists $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

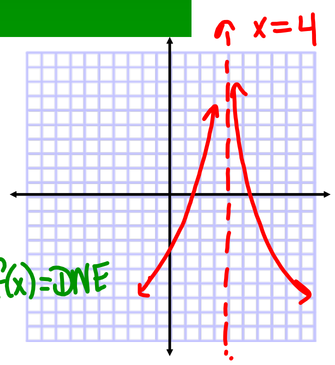
  3) limit as x approaches ***a*** of f(x) is equal to f(***a***)

Types of Discontinuities

- **Infinite** (essential): this type occurs near a vertical asymptote

  Justification: ***a*** is not in the domain of f(x) and the limit as x approaches ***a*** does not exist

Infinite/Essential Discontinuity @ x = 4



1) f(4) = undefined

2) $\lim_{x \to 4^-} f(x) = \infty$

$\lim_{x \to 4^+} f(x) = \infty$ $\therefore \lim_{x \to 4} f(x) = DNE$

3) $f(4) \neq \lim_{x \to 4} f(x)$

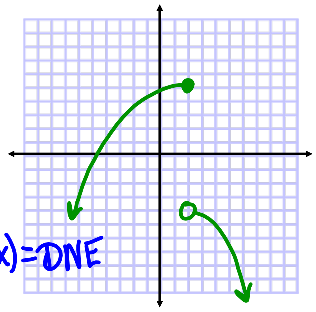
$\therefore$ Since f(4) is undefined and the left-hand & right-hand limits are infinite, then there is an essential/infinite discontinuity @ x=4.

Discontinuities cont.

- **Jump**: this occurs when the value of the function changes abruptly, in a non-smooth manner

Justification: the limit as x approaches *a* from the left is not equal to the limit as x approaches *a* from the right

Jump Discontinuity @ $x=2$



1) $f(2)=5$

2) $\lim_{x\to2^-} f(x)=5$

$\lim_{x\to2^+} f(x)=-4$ $\therefore \lim_{x\to2} f(x)=DNE$

3) $f(2)\neq\lim_{x\to2} f(x)$

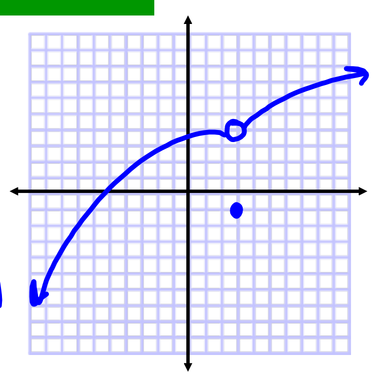
$\because$ Since $f(2)$ exists, but the left-hand and right-hand limits are not infinite and are not equal, there is a jump discontinuity @ $x=2$.

Discontinuities cont.

- **Point**: occurs when the graph is smooth and continuous except for a hole caused by a missing domain element that is replaced by a single coordinate elsewhere on the curve

Justification: the limit as x approaches *a* is not equal to f(*a*)

Point Discontinuity @ $x=3$



1) $f(3)=-1$

2) $\lim_{x\to3^-} f(x)=4$

$\lim_{x\to3^+} f(x)=4$ $\therefore \lim_{x\to3} f(x)=4$

3) $f(3)\neq$

Discontinuities cont.

"Hole"

- **Removable**: occurs when rational functions have common factors in the numerator and denominator

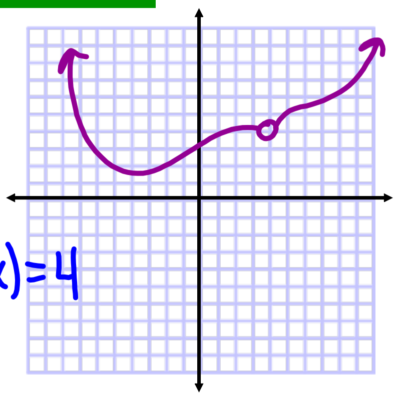
  Justification: f(*a*) does not exist, but the limit as x approaches *a* does

1) $f(4) = \text{undefined}$

2) $\lim_{x \to 4^-} f(x) = 4$

   $\lim_{x \to 4^+} f(x) = 4$ $\therefore \lim_{x \to 4} f(x) = 4$

3) $f(4) \neq \lim_{x \to 4} f(x)$



Discontinuities cont.

- **Domain Issue**: occurs when the domain is restricted on certain intervals

  Justification: the function is undefined for a set of values and no limits exist for the set of values, but do exist for any value in the domain excluding the endpoints

D: $\{x \mid x \leq -3, x \geq 4\}$

① $f(-3) = 0$

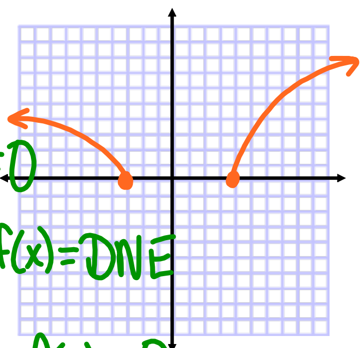
② $\lim_{x \to -3^-} f(x) = 0$

$\lim_{x \to -3^+} f(x) = \text{DNE}$

② $f(4) = 0$

$\lim_{x \to 4^-} f(x) = \text{DNE}$

$\lim_{x \to 4^+} f(x) = 0$



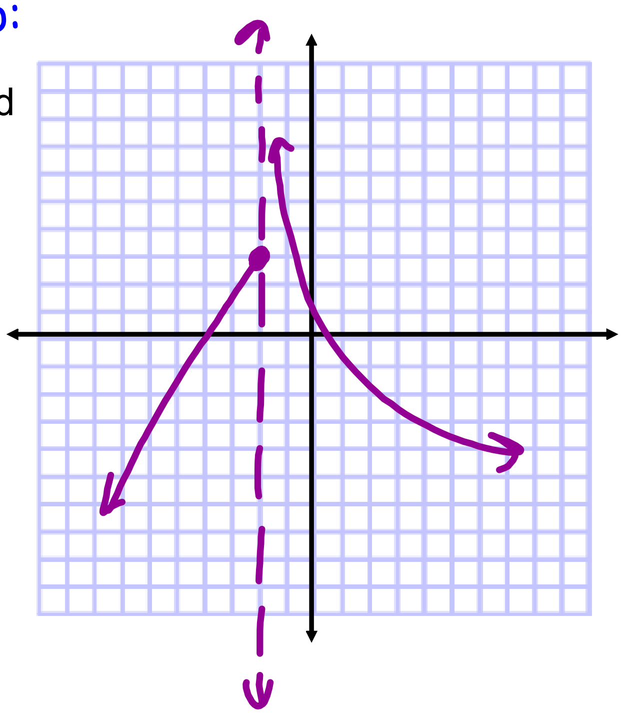
Infinite or Essential Jump:

f(a) exists, but either the left-hand or right-hand limit will be infinite

$f(-2)=3$

$\lim_{x\to -2^-} f(x)=3$

$\lim_{x\to -2^+} f(x)=\infty$



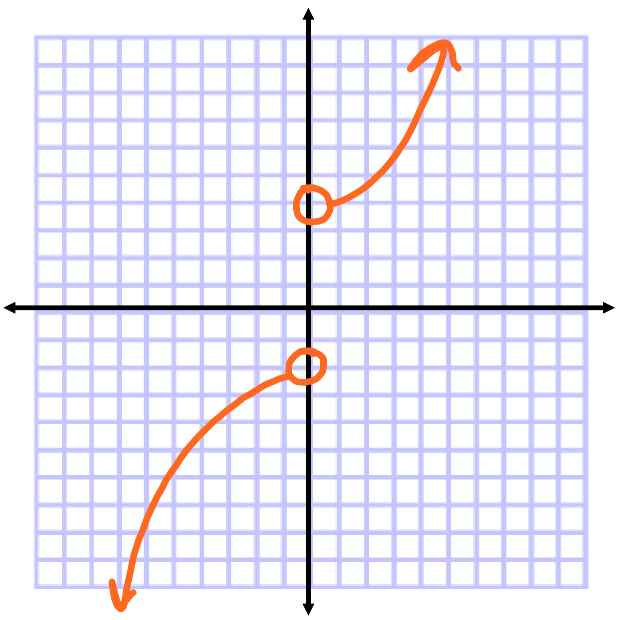
Removable Jump:

neither the left-hand or right-hand limit will be infinite

1) $f(0)=$ undefined

2) $\lim_{x\to 0^-} f(x)=-2$

$\lim_{x\to 0^+} f(x)=4$



Assignment:

Create the graph of a function that has at least 4 different types of discontinuities

- Name each type of discontinuity
- Justify (using the definition of continuity) how you know the type of discontinuity for each one