Today's Plan:

Learning Target (standard): I will calculate the determinants of 2 x 2 and 3 x 3 matrices and use them to solve a linear system with Cramer's Rule.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Determinants Practice:

1) D = 3
2) D = 20
3) D = -90
4) D = 69
5) D = -96
6) D = -123
7) D = -16
8) D = -27

Find the determinant of each matrix.

$$\begin{array}{c} \begin{matrix} + & \quad - & \quad + \end{matrix} \\ \begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} \end{array} = \begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 0 & -2 \end{vmatrix}$$

$$= (2+2) - 4(6-0) - 2(-6-0)$$

$$= 4 - 24 + 12$$

$$D = -8$$

Find the determinant of each matrix.

$$\begin{vmatrix} -2 & 4 & 2 \\ 3 & -3 & -4 \\ 1 & 0 & -2 \end{vmatrix} = -2\begin{vmatrix} -3 & -4 \\ 0 & -2 \end{vmatrix} - 4\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} + 2\begin{vmatrix} 3 & -3 \\ 1 & 0 \end{vmatrix}$$

$$= -2(6-0) - 4(-6+4) + 2(0+3)$$

$$= -12 + 8 + 6$$

$$D = 2$$

Find the determinant of each matrix.

$$\begin{vmatrix} 3 & -2 & 3 \\ -2 & 5 & 0 \\ 1 & -1 & -2 \end{vmatrix} = 3\begin{vmatrix} 5 & 0 \\ -1 & -2 \end{vmatrix} + 2\begin{vmatrix} -2 & 0 \\ 1 & -2 \end{vmatrix} + 3\begin{vmatrix} -2 & 5 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-10-0) + 2(4-0) + 3(2-5)$$

$$= -30 + 8 - 9$$

$$D = -31$$

Find the determinant of each matrix.

$$\begin{vmatrix} 4 & 3 & 1 \\ 3 & -2 & 3 \\ -1 & 1 & 5 \end{vmatrix} = 4\begin{vmatrix} -2 & 3 \\ 1 & 5 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 \\ -1 & 5 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ -1 & 1 \end{vmatrix}$$

$$= 4(-10-3) - 3(15+3) + 1(3-2)$$

$$= -52 - 54 + 1$$

$$D = -105$$

Find the determinant of each matrix.

$$\begin{vmatrix} -1 & -3 & -2 \\ 2 & 4 & -4 \\ -1 & 6 & 5 \end{vmatrix} = -1\begin{vmatrix} 4 & -4 \\ 6 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -4 \\ -1 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ -1 & 6 \end{vmatrix}$$

$$= -1(20+24) + 3(10-4) - 2(12+4)$$

$$= -44 + 18 - 32$$

$$D = -58$$

Cramer's Rule:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$D = \begin{array}{cc} x & y \\ \end{array}\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$ determinant of the system

$$D_x = \begin{array}{cc} = & y \\ \end{array}\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$ determinant of the x's

$$D_y = \begin{array}{cc} x & = \\ \end{array}\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$ determinant of the y's

Types of Solutions:

- Independent $D \neq 0$
  (x, y)
- Inconsistent $D = 0$ and at least one of either D_x or D_y is not 0
  no solution
- Dependent
  infinite solutions $D = 0 = D_x = D_y$

Solve the system using Cramer's Rule.

$$x + y = 3$$

$$3x - 2y = -6$$

independent

$(0, 3)$

$$D = \begin{array}{cc} x & y \\ \end{array}$$
$$D = \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -2 - 3$$

$D = -5$

$$D_x = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6 + 6$$

$D_x = 0$

$$D_y = \begin{vmatrix} 1 & 3 \\ 3 & -6 \end{vmatrix} = -6 - 9$$

$D_y = -15$

$$x = \frac{D_x}{D} = \frac{0}{-5}$$

$$y = \frac{D_y}{D} = \frac{-15}{-5}$$

Cramer's Rule for 3x3 Systems:

$$a_1x+b_1y+c_1z=d_1$$
$$a_2x+b_2y+c_2z=d_2$$
$$a_3x+b_3y+c_3z=d_3$$

$$D=\begin{array}{c}\begin{array}{ccc} x & y & z \end{array}\\ \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\end{array}$$

$$D_x=\begin{array}{c}\begin{array}{ccc} = & y & z \end{array}\\ \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}\end{array} \qquad D_y=\begin{array}{c}\begin{array}{ccc} x & = & z \end{array}\\ \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}\end{array} \qquad D_z=\begin{array}{c}\begin{array}{ccc} x & y & = \end{array}\\ \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}\end{array}$$

Solve the system using Cramer's Rule.

$$x+y+z=4$$
$$2x-y+z=-1$$
$$2x+y+z=3$$

$$D=\begin{array}{c}\begin{array}{ccc} x & y & z \end{array}\\ \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}\end{array}$$

$$=1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}+\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$=1(-1-1)-1(2-2)+(2+2)$$

$$=-2+0+4$$

$$D=2$$

$$D_x=\begin{array}{c}\begin{array}{ccc} = & y & z \end{array}\\ \begin{vmatrix} 4 & 1 & 1 \\ -1 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix}\end{array}$$

$$=4\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix}+1\begin{vmatrix} -1 & -1 \\ 3 & 1 \end{vmatrix}$$

$$=4(-1-1)-1(-1-3)+1(-1+3)$$

$$=-8+4+2$$

$$D_x=-2$$

$$x=\frac{D_x}{D}=\frac{-2}{2}$$

$$x=-1$$

$$D_y=\begin{array}{c}\begin{array}{ccc} x & = & z \end{array}\\ \begin{vmatrix} 1 & 4 & 1 \\ 2 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix}\end{array}$$

$$=1\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix}-4\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}+1\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix}$$

$$=1(-1-3)-4(2-2)+1(6+2)$$

$$=-4+0+8$$

$$D_y=4$$

$$y=\frac{D_y}{D}=\frac{4}{2}$$

$$y=2$$

$$D_z=\begin{array}{c}\begin{array}{ccc} x & y & = \end{array}\\ \begin{vmatrix} 1 & 1 & 4 \\ 2 & -1 & -1 \\ 2 & 1 & 3 \end{vmatrix}\end{array}=1\begin{vmatrix} -1 & -1 \\ 1 & 3 \end{vmatrix}-1\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix}+4\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}$$

$$=1(-3+1)-1(6+2)+4(2+2)$$

$$=-2-8+16$$

$$D_z=6$$

$$z=\frac{D_z}{D}=\frac{6}{2}$$

$$z=3$$

independent

(-1,2,3)

Assignment:

Cramer's Rule Worksheet #1-6

* show ALL required steps *