Today's Plan:

Learning Target (standard): I will calculate the determinants of 2 x 2 and 3 x 3 matrices.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Solutions to Elimination Method (3x3):

1) $independent(-3, 2, 1)$

2) $independent(5, 0, 3)$

3) $independent(-2, -1, -3)$

4) $inconsistent$ (no solution)

5) $independent(-3, 4, -4)$

6) $independent(1, -4, -1)$

Solve using the elimination method.

$4 \left(2x - y + 3z = 9 \right)$
$x + 4y + 4z = 5$
$-2 \left(3x + 2y + 2z = 5 \right)$

$8x - 4y + 12z = 36$
$x + 4y + 4z = 5$
$-6x - 4y - 4z = -10$

$9x + 16z = 41$
$-5x = -5$
$\boxed{x = 1}$

$9 + 16z = 41$
$16z = 32$
$\boxed{z = 2}$

$3 + 2y + 4 = 5$
$2y + 7 = 5$
$2y = -2$
$\boxed{y = -1}$

independent
$(1, -1, 2)$

Solve using the elimination method.

$2 \left(3x - 2y + z = 2 \right)$
$-1 \left(2x + 3y + 2z = -6 \right)$
$2 \left(3x - y + z = 0 \right)$

$6x - 4y + 2z = 4$
$-2x - 3y - 2z = 6$
$6x - 2y + 2z = 0$

$4x - 7y = 10$
$-1 \left(4x - 5y = 6 \right)$

$4x - 7y = 10$
$-4x + 5y = -6$
$-2y = 4$
$\boxed{y = -2}$

$4x + 10 = 6$
$4x = -4$
$\boxed{x = -1}$

$-3 + 2 + z = 0$
$-1 + z = 0$
$\boxed{z = 1}$

independent
$(-1, -2, 1)$

Matrix: A rectangular array of numbers

$$\begin{bmatrix} 1 & -8 & 3 \\ 5 & 6 & 7 \\ \dfrac{5}{2} & 0 & -2 \end{bmatrix}$$

← rows

elements

↑ columns

Element: Each number in the matrix

Rows: Horizontal numbers

Columns: Vertical numbers

Order of a Matrix: number of rows by the number of columns

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

3 rows × 3 columns

a_{ij}

row #   column #

a_{12} = "element in row 1, column 2"

Determinant of a 2 x 2 matrix:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\begin{vmatrix} 4 & -2 \\ 3 & 5 \end{vmatrix} = 4(5) - (-2)(3)$$
$$= 20 + 6$$
$$D = 26$$

Minor of a matrix: M_{ij}

- the determinant of a matrix after row *i* and column *j* have been removed

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \\ 2 & -5 & 0 \end{bmatrix}$$

$$M_{23} = \begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix} = 1(-5) - (-2)(2)$$
$$= -5 + 4$$
$$M_{23} = -1$$

$$M_{12} = \begin{vmatrix} -3 & 1 \\ 2 & 0 \end{vmatrix} = -3(0) - 1(2)$$
$$= 0 - 2$$
$$M_{12} = -2$$

Minor of a matrix:

- the determinant of a matrix after row *i* and column *j* have been removed

$$M_{ij}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \\ 2 & -5 & 0 \end{bmatrix}$$

$$M_{23} = \begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix} = -5 + 4$$

$$M_{23} = -1$$

$$M_{12} = \begin{vmatrix} -3 & 1 \\ 2 & 0 \end{vmatrix} = 0 - 2$$

$$M_{12} = -2$$

Cofactor of a matrix:

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$\begin{bmatrix} 2 & -5 & -2 \\ -3 & -1 & 0 \\ 1 & 4 & 4 \end{bmatrix}$$

$$C_{23} = (-1)^{2+3} M_{23}$$

$$= -1 \cdot M_{23}$$

$$= -1 \cdot \begin{vmatrix} 2 & -5 \\ -1 & 4 \end{vmatrix}$$

$$= -1 \cdot \left[2(4) - (-5)(-1) \right]$$

$$= -1(8 - 5)$$

$$C_{23} = -3$$

$$C_{33} = (-1)^{3+3} M_{33}$$

$$= 1 \cdot \begin{vmatrix} 2 & -5 \\ -3 & -1 \end{vmatrix} = 2(-1) - (-5)(-3)$$

$$= -2 - 15$$

$$C_{33} = -17$$

Find on 3x3

$$C_{11}, C_{12}, C_{13}$$

Cofactor of a matrix:

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$C_{13} = (-1)^{1+3} M_{13}$$

$$= M_{13}$$

$$\begin{bmatrix} 2 & -5 & -2 \\ -3 & 1 & 0 \\ -1 & 4 & -4 \end{bmatrix}$$

$$= \begin{vmatrix} -3 & -1 \\ -1 & 4 \end{vmatrix}$$

$$= -12 - 1$$

$$C_{21} = (-1)^{2+1} M_{21}$$

$$C_{13} = -13$$

$$= -M_{21}$$

$$= -\begin{vmatrix} -5 & -2 \\ 4 & -4 \end{vmatrix}$$

$$= -1(20 + 8)$$

$$C_{21} = -28$$

Assignment:

Determinants Worksheet #1-8

* show ALL steps *