

Today's Plan:

Learning Target (standard): I will determine the equations for tangent and normal lines.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

- Wkst 80 #2,3,5,7,9,10

$$2)c = \frac{4\sqrt{3}}{3}$$

$$3)c = \frac{-12 + 8\sqrt{3}}{3}$$

5) the function is not continuous at $x = 0$

$$7)c = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$$

9) the function is not continuous at $x = 0$

$$10)c = \frac{1}{8}$$

Determine whether the function satisfies the hypothesis of the MVT and if so, find c that satisfies the conclusion. Explain what this c represents.

$$f(x) = 2\sqrt{x} \quad \text{continuous } [1, 4] \checkmark$$

$$[1, 4]$$

$$f(x) = 2x^{\frac{1}{2}}$$

$$f'(x) = x^{-\frac{1}{2}}$$

$$f'(x) = \frac{1}{\sqrt{x}} \quad \text{differentiable } (1, 4) \checkmark$$

$$f(b) - f(a) = f'(c)(b - a)$$

$$f(4) - f(1) = \frac{1}{\sqrt{c}}(4 - 1)$$

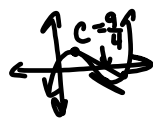
$$4 - 2 = 3c^{-\frac{1}{2}}$$

$$2 = 3c^{-\frac{1}{2}}$$

$$\frac{2}{3} = c^{-\frac{1}{2}}$$

$$(c^{\frac{1}{2}})^2 = \left(\frac{3}{2}\right)^2$$

$$c = \frac{9}{4}$$



A tangent line to $f(x) = 2\sqrt{x}$ through the point $x = \frac{9}{4}$ will be parallel to the secant line to $f(x) = 2\sqrt{x}$ through $x = 1$ and $x = 4$. In other words, the instantaneous rate of change at $x = \frac{9}{4}$ will be the same as the average rate of change between $x = 1$ and $x = 4$.

Determine whether the function satisfies the hypothesis of the MVT and if so, find c that satisfies the conclusion.

$$f(x) = x + \frac{1}{x} \quad \text{continuous } [-4, 4] \times$$

$$[-4, 4]$$

The function is
undefined @ $x = 0$, so
a "c" is not guaranteed
by the MVT.

Determine whether the function satisfies the hypothesis of the MVT and if so, find c that satisfies the conclusion.

$$f(x) = \frac{6}{x} - 3 \quad \text{Continuous } [1,2] \checkmark$$

$[1,2]$

$$f(x) = 6x^{-1} - 3$$

$$f'(x) = -6x^{-2} \quad \text{differentiable } (1,2) \checkmark$$

$$f'(x) = -\frac{6}{x^2}$$

$$f(b) - f(a) = (b-a)f'(c)$$

$$0 - 3 = (2-1)\left(-\frac{6}{c^2}\right)$$

$$-3 = 1\left(-\frac{6}{c^2}\right)$$

$$-3 = -\frac{6}{c^2}$$

$$-3c^2 = -6$$

$$\sqrt{c^2} = \sqrt{2}$$

$$c = \sqrt{2}, -\sqrt{2} \quad \leftarrow \text{not in } (1,2)$$

$$c = \sqrt{2}$$

Equations for Tangent & Normal Lines:

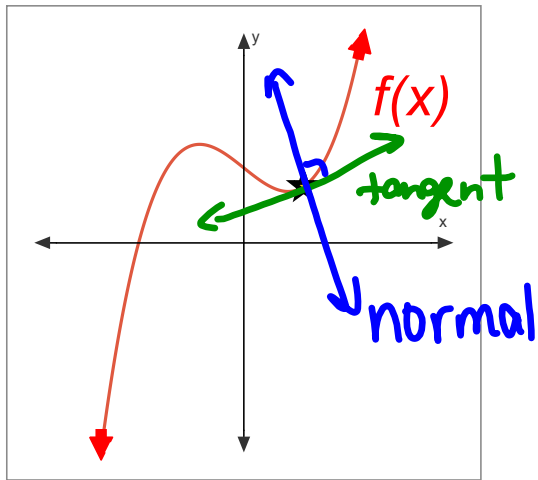
- the equation for any oblique line can always be written as $y = mx + b$
- a slope and an ordered pair are needed to write the equation
- the slope of a tangent line through a point on a curve is the value of the derivative at that point

$$m_{\tan(a, f(a))} = f'(a)$$

- the slope of a normal line through a point on a curve is the negative reciprocal value of the derivative at that point

$$m_{\text{normal}(a, f(a))} = -\frac{1}{f'(a)}$$

\therefore tangent \perp normal



Find the equation of the tangent line and the normal to the curve through the given point.

$$f(x) = 5x^2 \quad f'(x) = 10x$$

$$(3, 45)$$

$$m_{\text{tan}(3,45)} = 10(3) = 30$$

$$y = mx + b$$

$$45 = 30(3) + b$$

$$45 = 90 + b$$

$$b = -45$$

$$y = 30x - 45$$

$$-30x + y = -45$$

$$30x - y = 45$$

slope-intercept
 $y = mx + b$

standard
 $Ax + By = C$

↑ (+) ;
no fractions

Normal:

$$m_{\text{normal}(3,45)} = -\frac{1}{30}$$

$$y = mx + b$$

$$45 = -\frac{1}{30}(3) + b$$

$$45 = -\frac{1}{10} + b$$

$$b = \frac{451}{10}$$

$$y = -\frac{1}{30}x + \frac{451}{10}$$

$$\left(\frac{1}{30}x + y = \frac{451}{10} \right)$$

$$x + 30y = 1353$$

standard

slope-intercept

Find the values of a , b , and c where the curves

$$y_1 = x^2 + ax + b \quad \text{and} \quad y_2 = cx + x^2$$

have a common tangent line at $(-1, 0)$.

$$y_1' = 2x + a \quad y_1' = y_2'$$

$$y_2' = 2x + c \quad 2x + a = 2x + c$$

$$\boxed{a = c}$$

$(-1, 0)$ must be on both curves

$$y = ax^2 + bx + c \quad y = cx + x^2$$

$$0 = (-1)^2 + a(-1) + b \quad 0 = c(-1) + (-1)^2$$

$$0 = 1 - a + b \quad 0 = -c + 1$$

$$0 = 1 - 1 + b \quad -1 = -c$$

$$b = 0 \quad \boxed{c = 1}$$

$$\quad \quad \quad \boxed{a = 1}$$

Assignment:

- Wkst 75 #3, 4, 5, 8, 9, 11, 13, 14