Today's Plan:

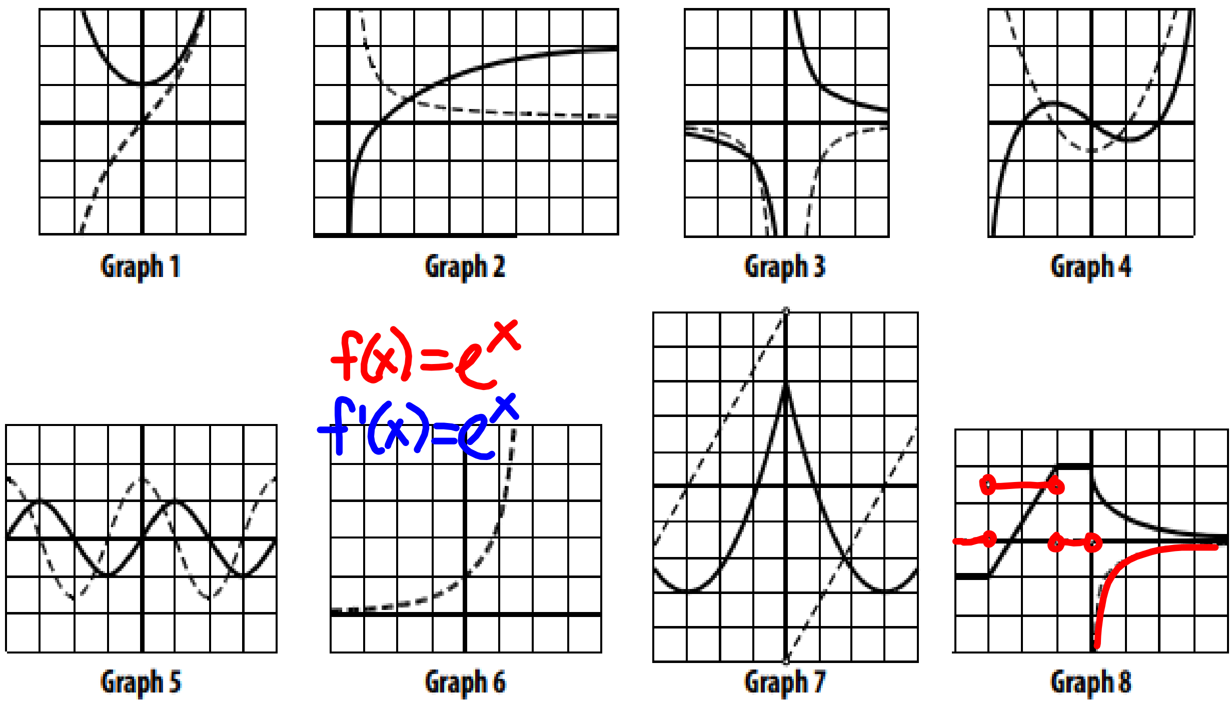
Learning Target (standard): I will decribe critical numbers and use them to find the exrtema of a function.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.



Find the derivative.

$$f(x)=\sqrt[3]{7x^2-4x+3}$$

$$f(x)=(7x^2-4x+3)^{\frac{1}{3}}$$

$$f'(x)=\tfrac{1}{3}(7x^2-4x+3)^{-\frac{2}{3}}(14x-4)$$

$$f'(x)=\frac{2(7x-2)}{3\sqrt[3]{(7x^2-4x+3)^2}}$$

Find the derivative.

$$f(x)=\frac{6}{(3x^2-1)^4}$$

$$f(x)=6(3x^2-1)^{-4}$$

$$f'(x)=-24(3x^2-1)^{-5}(6x)$$

$$f'(x)=\frac{-144x}{(3x^2-1)^5}$$

Find y'.

$$5x^3 - 2x^2y^2 + 4y^3 - 7 = 0$$

$$15x^2 - 4xy^2 - 4x^2yy' + 12y^2y' = 0$$

$$-4x^2yy' + 12y^2y' = 4xy^2 - 15x^2$$

$$y'(-4x^2y + 12y^2) = 4xy^2 - 15x^2$$

$$y' = \frac{4xy^2 - 15x^2}{12y^2 - 4x^2y}$$

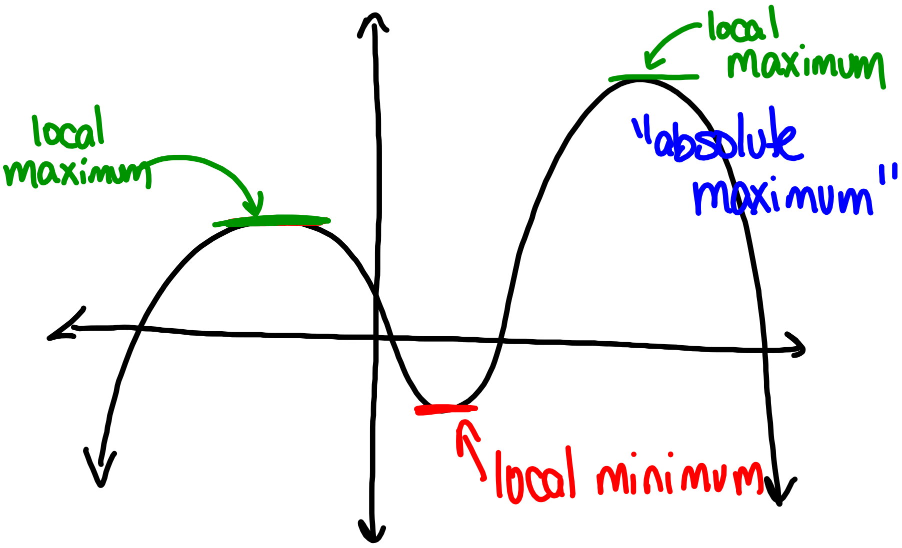
$$y' = \frac{x(4y^2 - 15x)}{4y(3y - x^2)}$$

Extrema of Functions:

- "high" points - maxima
- "low" points - minima

"relative"

- these are considered local or neighborhood extrema

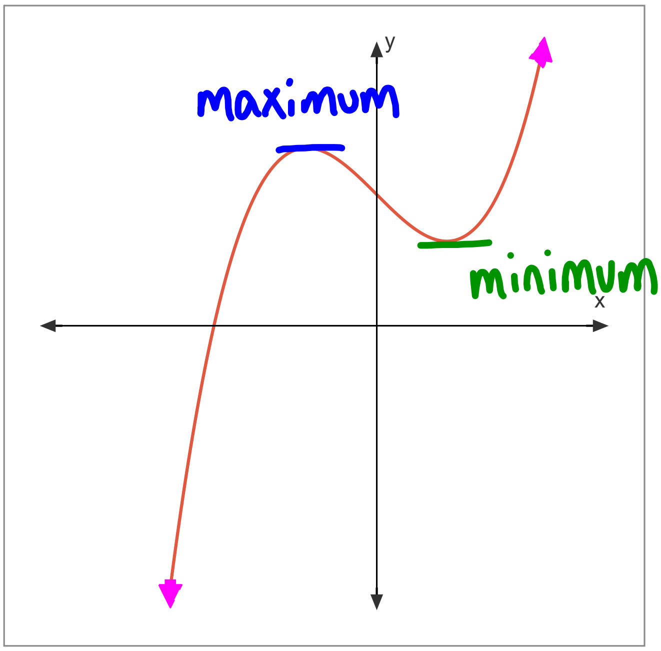


Definitions

$f'(x)=0$ or $f'(x)=\text{und}$

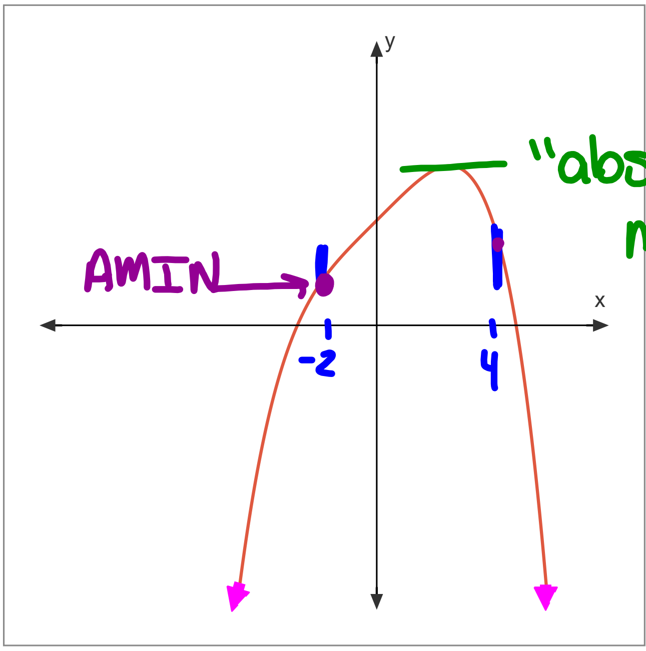
A **maximum** or **minimum** of a function occurs at a point where the derivative of a function is zero or undefined. "cusp"

A **relative (local)** maximum or minimum means that the curve has a horizontal or cusp tangent line at that point, but it is not the highest or lowest value that the function attains.



Definitions cont.

- An **absolute** maximum or minimum is a point where the curve has:
  - •a horizontal tangent line,
  - •a derivative that is undefined, or
  - •a point at an end of the domain of the function where the function attains its highest or lowest value.



"absolute maximum"

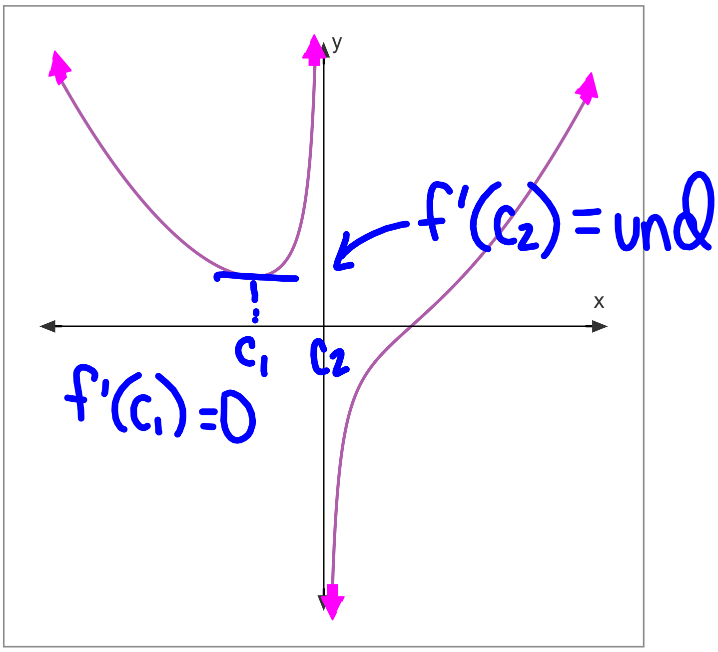
AMAX

[-2, 4]

Definitions cont.

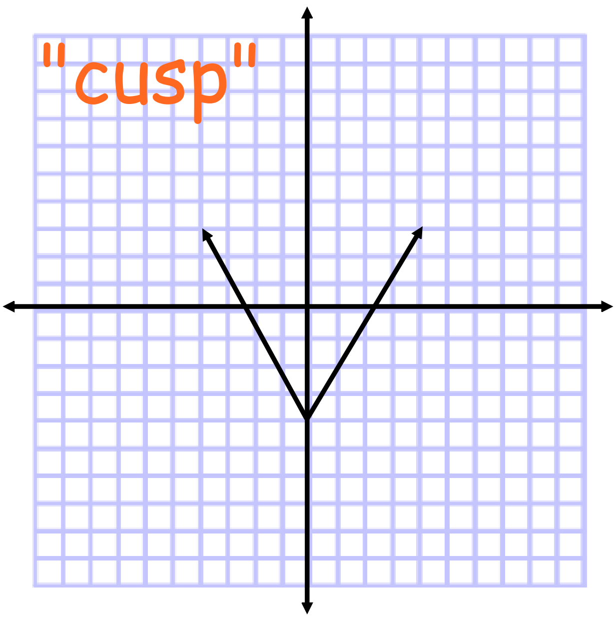
- A **critical number** is a number c in the domain of a function such that either $f'(c) = 0$ or $f'(c)$ is undefined or does not exist.

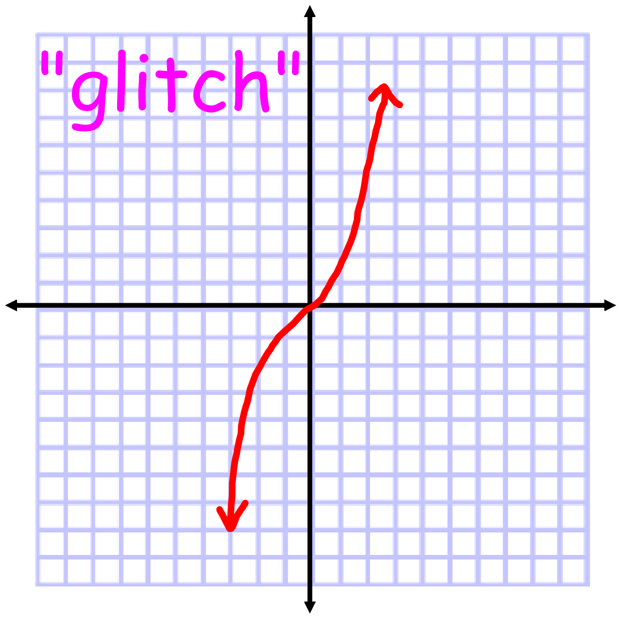
- A **critical point** is the coordinate $(c, f(c))$.



Other Extremum

- Cusp –a maximum or minimum that forms a "corner" and has an undefined derivative
- Glitch –a point that will produce a critical number but will not form an extrema





Determining Absolute Extrema

- First, find the critical numbers of f(x).
- Second, calculate f(c) for each critical number.
- Third, if f(x) is defined on a closed interval [a,b], calculate f(a) and f(b).
- Finally, determine the lowest and highest value.

Find the critical numbers.

$$f(x) = (x+5)^2\sqrt[3]{x-4}$$

$$f(x) = (x+5)^2(x-4)^{\frac{1}{3}}$$

$$f'(x) = 2(x+5)(x-4)^{\frac{1}{3}} + \frac{1}{3}(x-4)^{-\frac{2}{3}}(x+5)^2$$

$$= \frac{1}{3}(x+5)(x-4)^{-\frac{2}{3}}\left[6(x-4)+(x+5)\right]$$

$$= \frac{1}{3}(x+5)(x-4)^{-\frac{2}{3}}(6x-24+x+5)$$

$$f'(x) = \frac{(x+5)(7x-19)}{3\sqrt[3]{(x-4)^2}}$$

Critical #s:

$$x = -5, \frac{19}{7}, 4$$

Find the absolute extrema.

$$f(x) = x^3 - 12x; [-3, 5]$$

$f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x+2)(x-2)$

Critical #s:

$x = -2, 2$

$f(-2) = (-2)^3 - 12(-2)$

$= -8 + 24$

$f(-2) = 16$

$f(2) = 2^3 - 12(2)$

$= 8 - 24$

$f(2) = -16$

$f(-3) = (-3)^3 - 12(-3)$

$= -27 + 36$

$f(-3) = 9$

$f(5) = 5^3 - 12(5)$

$= 125 - 60$

$f(5) = 65$

$$\boxed{\begin{array}{l} \text{AMAX} = 65 \text{ @ } x = 5 \\ \text{AMIN} = -16 \text{ @ } x = 2 \end{array}}$$

Assignment:

p.139 #1-4, 9-14