

## Today's Plan:

**Learning Target (standard):** I will graph rational functions using the 7-step process.

**Students will:** Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

**Teacher will:** Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

**Assessment:** Board work, homework check and homework assignment

**Differentiation:** Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Please take a few minutes to go over your  
graphs with someone near you!



\* Quiz tomorrow! \*



Graph using transformations. Find the domain and range.

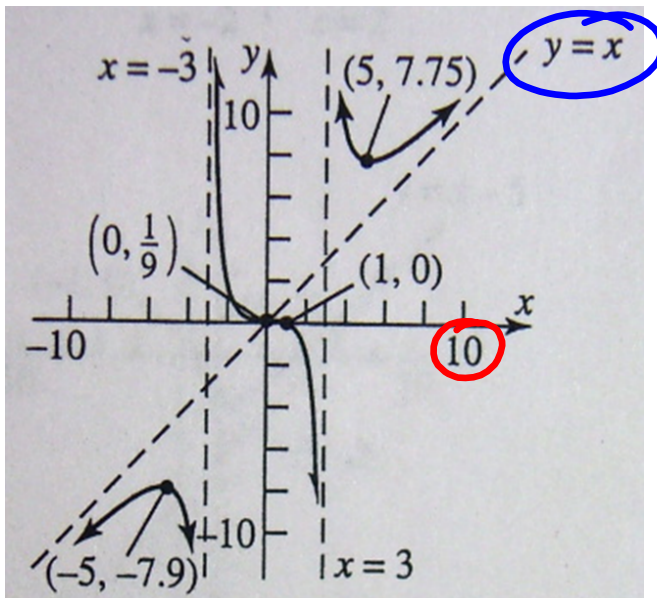
$$f(x) = \frac{2}{-\frac{1}{2}x + 3} - 4$$

$$f(x) = \frac{-4}{x-6} - 4$$

parent:  $f(x) = \frac{1}{x}$  VA:  $x=0$   
HA:  $y=0$

- 1)  $f(x) = -\frac{1}{x}$  r x
- 2)  $f(x) = -\frac{4}{x}$  v.s. by 4
- 3)  $f(x) = \frac{-4}{x-6}$  shift right 6  
VA:  $x=6$
- 4)  $f(x) = \frac{-4}{x-6} - 4$  shift down 4  
HA:  $y=-4$

D:  $\{x \mid x \neq 6\}$   
R:  $\{y \mid y \neq -4\}$



D:  $\{x \mid x \neq -3, 3\}$

R:  $\mathbb{R}$

VA:  $x = -3, x = 3$

HA: —

OA:  $y = x$

Holes: —

Graph using the 7-Step Process:

$$f(x) = \frac{x^2 + 3x - 10}{x^2 + 8x + 15} \quad f(x) = \frac{(x+5)(x-2)}{(x+3)(x+5)}$$

① D:  $\{x \mid x \neq -5, -3\}$

② I<sub>x</sub>: (2, 0)  
I<sub>y</sub>: (0, - $\frac{2}{3}$ )

③ Symmetry:  
 $f(-x) = \frac{-x-2}{-x+3}$   
 $\therefore$  neither  
 $f(-x) \neq f(x)$   
 $f(-x) \neq -f(x)$

④ Undefined Behavior:  
VA:  $x = -3$   
Hole:  $(-5, \frac{7}{2})$   
 $f(-5) = \frac{-5-2}{-5+3} = \frac{-7}{-2} = \frac{7}{2}$

⑤ End Behavior:  
HA:  $y = 1$   
OA: - does not intersect  
EB: - intersect

intersects?  
 $1 = \frac{x-2}{x+3}$   
 $x+3 = x-2$   
 $3 \neq -2$

x-2	-	:	-	:	0	+
x+3	-	0	+	:	+	+
test	-4	-3		0	1	2
point	(-4, 6)		(-1, $\frac{1}{2}$ )		(3, $\frac{1}{6}$ )	
f(x)	above		below			above

Graph using the 7-Step Process:

$$f(x) = \frac{x^4 + 1}{x^2}$$

① D:  $\{x \mid x \neq 0\}$

② I<sub>x</sub>: -  
I<sub>y</sub>: -

③ Symmetry:  
 $f(-x) = \frac{(-x)^4 + 1}{(-x)^2} = \frac{x^4 + 1}{x^2} = f(x)$   
 $\therefore$  even

④ Undefined Behavior:  
VA:  $x = 0$   
Holes: -

⑤ End Behavior:  
HA: - intersects?  
OA: -  
EB:  $y = x^2$  does not intersect  
 $x^2 \sqrt{x^4 + 1}$   
 $x^2 = \frac{x^4 + 1}{x^2}$   
 $x^4 = x^4 + 1$   
 $0 \neq 1$

x <sup>4</sup> +1	+	:	+
x <sup>2</sup>	+	0	+
test	-1	0	1
point	(-1, 2)		(1, 2)
f(x)	above		above

y	x <sup>2</sup>
4	-2
1	-1
0	0
1	1
4	2

# Assignment:

p.237 #50, 62, 64, 66, 72

\* Quiz tomorrow! \*

Interval	Test Number	Value of $Q$	Graph of $Q$
$x < -2$	-3	$Q(-3) = 16$	Above x-axis
$-2 < x < -1$	-1.5	$Q(-1.5) \approx -2.3$	Below x-axis
$-1 < x < 1$	0	$Q(0) = \frac{1}{4}$	Above x-axis
$1 < x < 2$	1.5	$Q(1.5) \approx -2.3$	Below x-axis
$x > 2$	3	$Q(3) = 16$	Above x-axis

7.

$$Q\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^4 - 1}{\left(\frac{1}{2}\right)^2 - 4} = \frac{\frac{1}{16} - 1}{\frac{1}{4} - 4} = \frac{-\frac{15}{16}}{-\frac{15}{4}} = \frac{1}{4}$$

52.  $G(x) = \frac{x^3 + 1}{x^2 + 2x} = \frac{(x+1)(x^2 - x + 1)}{x(x+2)}$

- Domain:  $\{x \mid x \neq -2, x \neq 0\}$
- The x-intercept is -1. There is no y-intercept.
- Because  $G(-x) = \frac{-x^3 + 1}{x^2 - 2x}$ , we conclude that  $G$  is neither even nor odd.
- The vertical asymptotes are  $x = 0$  and  $x = -2$ .
- The line  $y = x - 2$  is an oblique asymptote; intersected at  $\left[-\frac{1}{4}, -\frac{9}{4}\right]$ .
- The zero of the numerator, -1, and the zeros of the denominators, 0 and -2, divide the x-axis into:  $x < -2$ ,  $-2 < x < -1$ ,  $-1 < x < 0$ ,  $x > 0$ .

Interval	Test Number	Value of $G$	Graph of $G$
$x < -2$	-3	$G(-3) = -8.75$	Below x-axis
$-2 < x < -1$	-1.5	$G(-1.5) = 3.2$	Above x-axis
$-1 < x < 0$	-0.5	$G(-0.5) = -1.2$	Below x-axis
$x > 0$	1	$G(1) = 0.67$	Above x-axis

7.

62.  $F(x) = \frac{x^2 + 3x + 2}{x - 1} = \frac{(x + 2)(x + 1)}{(x - 1)}$

- Domain:  $\{x \mid x \neq 1\}$
- The x-intercepts are -2 and -1. The y-intercept is  $F(0) = -2$ .
- Because  $F(-x) = \frac{x^2 - 3x + 2}{-x - 1}$ , we conclude  $F$  is neither even nor odd.
- The vertical asymptote is  $x = 1$ .
- The line  $y = x + 4$  is an oblique asymptote; not intersected.
- The zeros of the numerator, -2, and -1, and the zero of the denominator, 1, divide the x-axis into:  $x < -2$ ,  $-2 < x < -1$ ,  $-1 < x < 1$ ,  $x > 1$ .

Interval	Test Number	Value of $F$	Graph of $F$
$x < -2$	-3	$F(-3) = -0.5$	Below x-axis
$-2 < x < -1$	-1.5	$F(-1.5) = 0.1$	Above x-axis
$-1 < x < 1$ ( $\frac{1}{2}, \frac{1}{2}$ )	0	$F(0) = -2$	Below x-axis
$x > 1$	2	$F(2) = 12$	Above x-axis

7.

64.  $R(x) = \frac{x^2 - x - 12}{x + 5} = \frac{(x - 4)(x + 3)}{(x + 5)}$

- Domain:  $\{x \mid x \neq -5\}$
- The x-intercepts are 4 and -3. The y-intercept is  $R(0) = \frac{-12}{5}$ .
- Because  $R(-x) = \frac{x^2 + x - 12}{-x + 5}$ , we conclude  $R$  is neither even nor odd.
- The vertical asymptote is  $x = -5$ .
- The line  $y = x - 6$  is an oblique asymptote; not intersected.
- The zeros of the numerator and the zero of the denominator, divide the x-axis into:  $-5 < x < -3$ ,  $-3 < x < 4$ ,  $x > 4$ .

Interval	Test Number	Value of $R$	Graph of $R$
$x < -5$	-6	-30	Below x-axis
$-5 < x < -3$	-4	8	Above x-axis
$-3 < x < 4$	1	-2	Below x-axis
$x > 4$	5	4/5	Above x-axis

7.

66.  $G(x) = \frac{x^2 - x - 12}{x + 1} = \frac{(x - 4)(x + 3)}{(x + 1)}$

- Domain:  $\{x \mid x \neq -1\}$
- The x-intercepts are 4 and -3. The y-intercept is  $G(0) = -12$ .
- Because  $G(-x) = \frac{x^2 + x - 12}{-x + 1}$ , we conclude  $G$  is neither even nor odd.



66.  $G(x) = \frac{x^2 - x - 12}{x + 1} = \frac{(x - 4)(x + 3)}{(x + 1)}$

- Domain:  $\{x \mid x \neq -1\}$
- The x-intercepts are 4 and -3. The y-intercept is  $G(0) = -12$ .
- Because  $G(-x) = \frac{x^2 + x - 12}{-x + 1}$ , we conclude  $G$  is neither even nor odd.
- The vertical asymptote is  $x = -1$ .
- The line  $y = x - 2$  is an oblique asymptote; not intersected.
- The zeros of the numerator and the zero of the denominator divide the x-axis into:  $x < -3$ ,  $-3 < x < -1$ ,  $-1 < x < 4$ ,  $x > 4$ .

Interval	Test Number	Value of $G$	Graph of $G$
$x < -3$	-4	-8/3	Below x-axis
$-3 < x < -1$	-2	6	Above x-axis
$-1 < x < 4$	2	-10/3	Below x-axis
$x > 4$	5	4/3	Above x-axis

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7.

68.  $R(x) = \frac{x^3 - 3x^2 + x^2 - 2x + 6}{(x - 1)(x + 2)(x - 3)} = \frac{x^3 - 2x^2 - 2x + 6}{(x - 1)(x + 2)(x - 3)}$

- Domain:  $\{x \mid x \neq 0, x \neq 4\}$
- The x-intercepts are 1, -2 and 3. There is no y-intercept.  $6x^2 - 21x + 6 = 0$
- Because  $R(-x) = \frac{-x^3 - 2x^2 - 2x + 6}{(-x - 1)(-x + 2)(-x - 3)}$ , we conclude  $R$  is neither even nor odd.
- The vertical asymptotes are  $x = 0$  and  $x = 4$ .
- Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is  $y = 1$ ; intersected at  $(\frac{7 + \sqrt{53}}{4}, 1)$  and  $(\frac{7 - \sqrt{53}}{4}, 1)$ .
- The zeros of the numerator and the zeros of the denominator, divide the x-axis into:  $x < -2$ ,  $-2 < x < 0$ ,  $0 < x < 1$ ,  $1 < x < 3$ ,  $3 < x < 4$ ,  $x > 4$ .

Interval	Test Number	Value of $R$	Graph of $R$
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72.  $R(x) = \frac{8x^2 + 26 + 15}{2x^2 - x - 15} = \frac{(2x + 5)(4x + 3)}{(2x + 5)(x - 3)} = \frac{4x + 3}{x - 3}, x \neq \frac{-5}{2}$

Step 0:  $n = 2$   
 $m = 2$

Step 1: Domain:  $\{x \mid x \neq \frac{-5}{2}, x \neq 3\}$

Step 2: (a) x-intercept:  $\frac{-3}{4}$   
(b) y-intercept: -1

Step 3: No symmetry

Step 4: The vertical asymptote is  $x = 3$ . There is a hole at  $(\frac{-5}{2}, \frac{14}{11})$ .

Step 5: Since  $n = m$ , the horizontal asymptote is  $y = \frac{8}{2} = 4$ .

$$4 = \frac{4x + 3}{x - 3}$$

$$4x - 12 = 4x + 3$$

No solution, so horizontal asymptote is not intersected.

Step 6:

Interval	Test Number	$R(x)$	Graph of $R$
$x < \frac{-5}{2}$	-3	3	Above x-axis
$\frac{-5}{2} < x < \frac{-3}{4}$	-1	1	Above x-axis
$\frac{-3}{4} < x < 3$	0	-1	Below x-axis
$x > 3$	4	19	Above x-axis

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