Today's Plan:

Learning Target (standard): I will evaluate and describe infinite limits. I will analyze asymptotic behavior. I will use each of these to graph rational functions.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

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1) $^5/_2$	7) 0
2) $^1/_2$	8) 0
3) $-^7/_3$	9) $^5/_2$
4) $^3/_2$	10) $^3/_2$
5) 1	11) -1
6) 4	12) 0

5) $\lim_{x\to\infty} \sqrt[3]{\frac{8+x^2}{x^2+x}} = \lim_{x\to\infty} \sqrt[3]{\frac{\frac{8}{x^2}+\frac{x^2}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}}}$

$= \lim_{x\to\infty} \sqrt[3]{\frac{\frac{8}{x^2}+1}{1+\frac{1}{x}}}$

$= \sqrt[3]{\frac{0+1}{1+0}}$

$= \sqrt[3]{1}$

$= 1$

11) $\lim_{x\to-\infty} \frac{1+\sqrt[5]{x}}{1-\sqrt[5]{x}} = \lim_{x\to-\infty} \frac{1+x^{\frac{1}{5}}}{1-x^{\frac{1}{5}}}$

$= \lim_{x\to-\infty} \frac{\frac{1}{x^{\frac{1}{5}}}+\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}}}{\frac{1}{x^{\frac{1}{5}}}-\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}}}$

$= \lim_{x\to-\infty} \frac{\frac{1}{x^{\frac{1}{5}}}+1}{\frac{1}{x^{\frac{1}{5}}}-1}$

$= \frac{0+1}{0-1}$

$= -1$

6) $$\lim_{x\to-\infty} \frac{4x-3}{\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{\frac{4x}{x}-\frac{3}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{4-\frac{3}{x}}{\sqrt{1+\frac{1}{x^2}}}$$

$$= \frac{4-0}{\sqrt{1+0}}$$

$$= 4$$

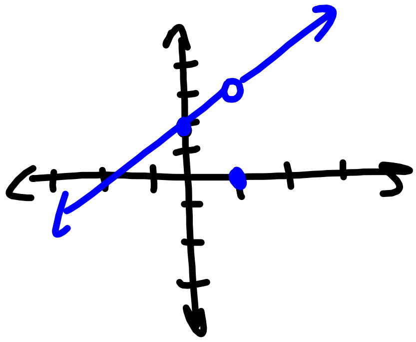
Suppose $f(x) = x+2$ if $x \neq 1$ and $f(\overset{x}{1}) = \overset{y}{0}$.

$$f(x) = \begin{cases} x+2, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

$\lim_{x\to1^-} f(x) = 3$

$\lim_{x\to1^+} f(x) = 3$

$\lim_{x\to1} f(x) = 3$



Evaluate:

$$\lim_{x\to\infty}\frac{\sqrt{x^2-4x+5}}{x^2} = \lim_{x\to\infty}\frac{\sqrt{\frac{x^2}{x^4}-\frac{4x}{x^4}+\frac{5}{x^4}}}{\frac{x^2}{x^2}}$$

$$\sqrt{?} = x^2$$

$$= \lim_{x\to\infty}\frac{\sqrt{\frac{1}{x^2}-\frac{4}{x^3}+\frac{5}{x^4}}}{1}$$

$$= \frac{\sqrt{0-0+0}}{1} = \frac{0}{1} = 0$$

Evaluate:

$$\lim_{x\to\infty}\left(\frac{1}{x}-\frac{x}{x-1}\right) = \lim_{x\to\infty}\left[\frac{x-1}{x(x-1)} - \frac{x^2}{x(x-1)}\right]$$

$$= \lim_{x\to\infty}\frac{-x^2+x-1}{x^2-x}$$

$$= \lim_{x\to\infty}\frac{\frac{-x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}}$$

$$= \lim_{x\to\infty}\frac{-1+\frac{1}{x}-\frac{1}{x^2}}{1-\frac{1}{x}}$$

$$= \frac{-1+0-0}{1-0}$$

$$= -1$$

Evaluate:

$$\lim_{x\to\infty}\left(\sqrt{x^2+7x}-x\right)\cdot\frac{\sqrt{x^2+7x}+x}{\sqrt{x^2+7x}+x}$$

$$=\lim_{x\to\infty}\frac{x^2+7x-x^2}{\sqrt{x^2+7x}+x}$$

$$=\lim_{x\to\infty}\frac{7x}{\sqrt{x^2+7x}+x}$$

$$\sqrt{x^2}=x$$

$$=\lim_{x\to\infty}\frac{\frac{7x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{7x}{x^2}}+\frac{x}{x}}$$

$$=\lim_{x\to\infty}\frac{7}{\sqrt{1+\frac{7}{x}}+1}$$

$$=\frac{7}{\sqrt{1+0}+1}$$

$$=\frac{7}{2}$$

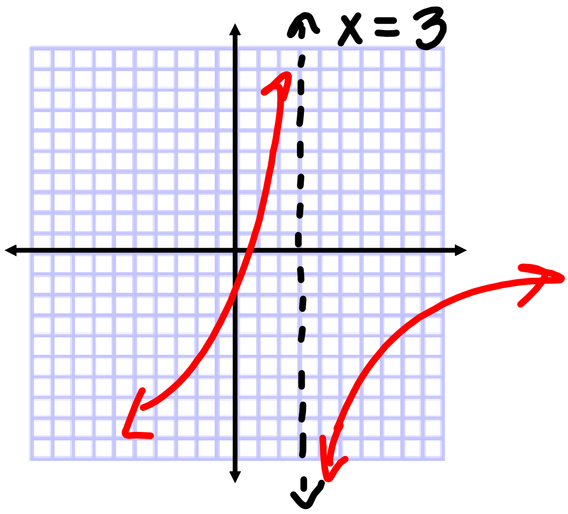
Rational Functions & Calculus:

- One-sided limits around vertical asymptotes determine:
  - infinite behavior around the vertical asymptotes
  - left-hand behavior does NOT need to be the same as the right-hand behavior

VA: $x=3$

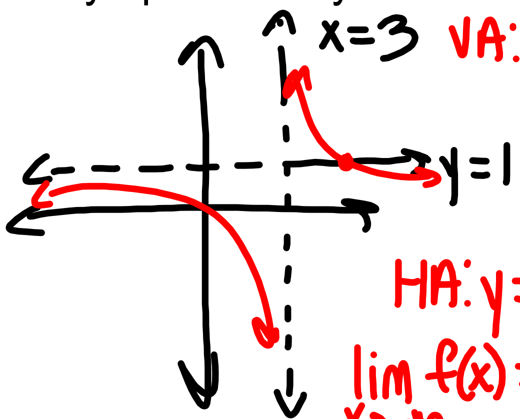
$$\lim_{x\to 3^-} f(x)=\infty$$

$$\lim_{x\to 3^+} f(x)=-\infty$$



Rational Functions & Calculus:

- Limits as *x* approaches infinity or negative infinity determine:
  - end behavior of a function
  - horizontal and oblique asymptotes may be intersected



VA: $x=3$

HA: $y=1$

$\lim_{x\to-\infty} f(x)=1$

$\lim_{x\to\infty} f(x)=1$

Rational Functions & Calculus:

- Determine the domain and any holes that may exist
- Find the *x* & *y*-intercepts
- Determine the horizontal or oblique asymptote, if it exists, and check to see if it is intersected *end behavior

$$\lim_{x\to\infty} f(x) \quad \& \quad \lim_{x\to-\infty} f(x)$$

- Determine the vertical asymptote(s) and the behavior around it/them *one-sided limits
- Sketch the graph

Graph:

$$f(x)=\frac{2x^2}{9-x^2}=\frac{2x^2}{(3+x)(3-x)}$$

① D: $\{x \mid x \neq -3, 3\}$

Holes: —

② I_x: $(0, 0)$ $\quad 0=\frac{2x}{9-x^2}$ $\quad 0=2x$

I_y: $(0, 0)$ $\quad f(0)=\frac{2(0)}{9-0^2}$

③ End Behavior:

$$\lim_{x\to\infty}\frac{2x^2}{9-x^2}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}}{\frac{9}{x^2}-\frac{x^2}{x^2}}=\lim_{x\to\infty}\frac{2}{\frac{9}{x^2}-1}=\frac{2}{0-1}=-2$$

$$\lim_{x\to-\infty}\frac{2}{\frac{9}{x^2}-1}=\frac{2}{0-1}=-2$$

$\therefore$ HA: $y=-2$

intersected?

$\frac{-2}{1}=\frac{2x^2}{9-x^2}$

$-18+2x^2=2x^2$

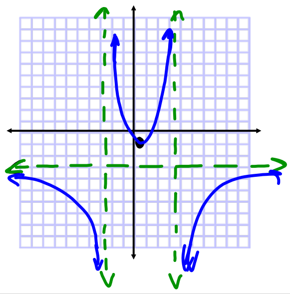
$-18\neq 0$

DNI

④ VA: $x=-3, x=3$

$\lim_{x\to-3^-} f(x)=-\infty$ $\qquad \lim_{x\to3^-} f(x)=\infty$

$\lim_{x\to-3^+} f(x)=\infty$ $\qquad \lim_{x\to3^+} f(x)=-\infty$



Assignment:

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Show ALL steps