

Today's Plan:

Learning Target (standard): I will find the average velocity and instantaneous velocity of a particle moving along a curve.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Slope of Tangent Lines Worksheet:

$$\begin{array}{lll}
 1) m_{\tan(a, f(a))} = 4a & 4) m_{\tan(a, f(a))} = -\frac{2}{a^2} & 7) m_{\tan(a, f(a))} = 9a^2 + 1 \\
 m_{\tan(2, 12)} = 8 & m_{\tan(1, 5)} = -2 & m_{\tan(1, 0)} = 10 \\
 2) m_{\tan(a, f(a))} = 4 & 5) m_{\tan(a, f(a))} = \frac{1}{\sqrt{2a}} & 8) m_{\tan(a, f(a))} = -3a^2 + 4a + 1 \\
 m_{\tan(1, -2)} = 4 & & m_{\tan(0, -3)} = 1 \\
 3) m_{\tan(a, f(a))} = -3a^2 + 3 & m_{\tan(2, 5)} = \frac{1}{2} & \\
 m_{\tan(1, 2)} = 0 & 6) m_{\tan(a, f(a))} = 2a + 2 & \\
 & m_{\tan(0, 1)} = 2 &
 \end{array}$$

$$5) f(x) = (2x)^{0.5} + 3$$



$$\begin{aligned} m_{\tan(a, f(a))} &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[\sqrt{2a+2h} + 3] - [\sqrt{2a} + 3]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2a+2h} - \sqrt{2a}}{h} \cdot \frac{\sqrt{2a+2h} + \sqrt{2a}}{\sqrt{2a+2h} + \sqrt{2a}} \\ &= \lim_{h \rightarrow 0} \frac{2a+2h-2a}{h(\sqrt{2a+2h} + \sqrt{2a})} \\ &= \lim_{h \rightarrow 0} \frac{2}{\sqrt{2a+2h} + \sqrt{2a}} \\ &= \frac{2}{\sqrt{2a} + \sqrt{2a}} \\ &= \frac{2}{2\sqrt{2a}} \end{aligned}$$

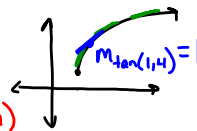
$$m_{\tan(2,5)} = \frac{1}{\sqrt{2 \cdot 2}}$$

$$m_{\tan(2,5)} = \frac{1}{2}$$

$$m_{\tan(a, f(a))} = \frac{1}{\sqrt{2a}}$$

$$f(x) = 2\sqrt{x} + 2$$

$$m_{\tan(1,4)} =$$



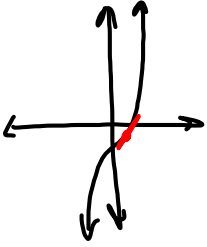
$$\begin{aligned} m_{\tan(a, f(a))} &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2\sqrt{a+h} + 2] - [2\sqrt{a} + 2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{2\sqrt{a+h} - 2\sqrt{a}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2\sqrt{a+h} - 2\sqrt{a}}{h} \cdot \frac{2\sqrt{a+h} + 2\sqrt{a}}{2\sqrt{a+h} + 2\sqrt{a}} \\ &= \lim_{h \rightarrow 0} \frac{4a+4h-4a}{h(2\sqrt{a+h} + 2\sqrt{a})} \\ &= \lim_{h \rightarrow 0} \frac{4}{2\sqrt{a+h} + 2\sqrt{a}} \\ &= \frac{4}{2\sqrt{a} + 2\sqrt{a}} \\ &= \frac{4}{4\sqrt{a}} \end{aligned}$$

$$m_{\tan(1,4)} = \frac{1}{\sqrt{1}} = 1$$

$$m_{\tan(a, f(a))} = \frac{1}{\sqrt{a}}$$

$f(x) = 3x^3 - 1$ Write the meaning of the general answer.

$m_{\tan(1,2)} =$



$$m_{\tan(a,f(a))} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[3(a+h)^3 - 1] - [3a^3 - 1]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(a^3 + 3a^2h + 3ah^2 + h^3) - 1 - 3a^3 + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{3a^3} + 9a^2h + 9ah^2 + 3h^3 - \cancel{1} - \cancel{3a^3} + \cancel{1}}{h}$$

$$= \lim_{h \rightarrow 0} (9a^2 + 9ah + 3h^2)$$

$m_{\tan(a,f(a))} = 9a^2$

$m_{\tan(1,2)} = 9(1)^2 = 9$

The slope of the tangent line through $(a, f(a))$ to $f(x) = 3x^3 - 1$ is $9a^2$.

Velocity

- Suppose a car leaves Cincinnati at 11:00am and at 1:00pm it stops 250 miles from Cincinnati. The car then reaches its destination of Knoxville (400 miles) at 4:00pm. What was the car's average velocity between the two stops?
- What was the car's velocity at 2:30pm?

$$d = rt$$

$$r = \frac{d}{t}$$

$$r = \frac{\Delta d}{\Delta t}$$

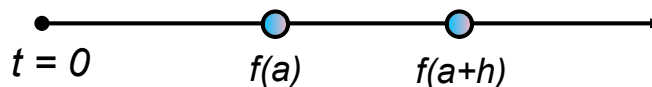
$$r = \frac{400 - 250}{5 - 2} = \frac{150}{3} = 50 \text{ mph}$$

$$\text{Average Velocity (AROC)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Velocity

- Average velocity means the car's speedometer would have obtained that velocity at some point during the trip, but did not necessarily travel at that rate for the entire trip

Position Function:



- After any certain time a , the particle has moved $f(a)$ units from where it started
- After any certain time $a+h$, the particle has moved $f(a+h)$ units from where it started

What would the particle's average velocity be on this time interval?

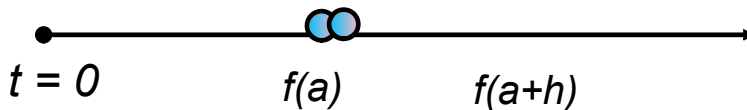
$$\text{Avg Vel} = \frac{\Delta d}{\Delta t} = \frac{f(a+h) - f(a)}{a+h - a}$$

$$\text{AVOC} = \frac{f(a+h) - f(a)}{h}$$

- Instantaneous velocity is the velocity the speedometer reads at a particular time

"velocity"

Position Function:



- After any certain time a , the particle has moved $f(a)$ units from where it started
- After any certain time $a+h$, the particle has moved $f(a+h)$ units from where it started

What would the particle's velocity be at $t = a$?

$$v(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Velocity:

- the sign on the velocity determines the direction of movement
- when the velocity is 0, the particle has momentarily stopped

Assignment:

Velocity Worksheet
#1-6