Today's Plan:

Learning Target (standard): I will find the inverse of a matrix and use matrix multiplication to verify the result.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Matrix Operations:

1) $\begin{bmatrix} 6 & 11 & 6 \end{bmatrix}$

2) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

3) $\begin{bmatrix} 40 & -20 \\ 20 & -40 \\ -40 & 40 \end{bmatrix}$

4) $\begin{bmatrix} 0 & -80u & 80u \end{bmatrix}$

5) $\begin{bmatrix} -26 & -4 & 12 \\ -61 & 31 & 12 \end{bmatrix}$

6) $\begin{bmatrix} 33 & 39 \\ 33 & -15 \end{bmatrix}$

7) $\begin{bmatrix} 36 & 24 & -72 \\ -51 & 36 & 12 \end{bmatrix}$

8) $\begin{bmatrix} 7 & 3 \\ -37 & 33 \\ 17 & -21 \end{bmatrix}$

9) $\begin{bmatrix} 0 & 0 \\ -30 & 10 \\ -30 & 10 \end{bmatrix}$

10) $\begin{bmatrix} 4 & -8 \\ 10 & -28 \\ 12 & -48 \end{bmatrix}$

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Solve using the matrix method.

$$-2x-2y=-10$$
$$5x+5y=-10$$

$-5 \quad -5 \quad -25$

$$\left[\begin{array}{cc|c} -2 & -2 & -10 \\ 5 & 5 & -10 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 5 & 5 & -10 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 0 & -35 \end{array}\right]$$

$0 \neq -35$

inconsistent

no solution

Solve using the matrix method.

$$2x-y+3z=9$$
$$x+4y+4z=5$$
$$3x+2y+2z=5$$

$-2 \quad -8 \quad -8 \quad -10$

$-3 \quad -12 \quad -12 \quad -15$

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 2 & -1 & 3 & 9 \\ 3 & 2 & 2 & 5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & -9 & -5 & -1 \\ 3 & 2 & 2 & 5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & -9 & -5 & -1 \\ 0 & -10 & -10 & -10 \end{array}\right]$$

$0 \quad 9 \quad 9 \quad 9$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & -10 & -10 & -10 \\ 0 & -9 & -5 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -9 & -5 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x+4y+4z=5$

$y+z=1$

$z=2$

$y+2=1$

$y=-1$

$x-4+8=5$

$x+4=5$

$x=1$

independent

$(1,-1,2)$

Find the product if it exists.

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -6-2 & 10-1 \\ -9+8 & 15+4 \end{bmatrix}$$

$$(2\times\underbrace{2)(2}_{=\checkmark}\times 2) = 2\times 2$$

$$= \begin{bmatrix} -8 & 9 \\ -1 & 19 \end{bmatrix}$$

Find the product if it exists.

$$\begin{bmatrix} 3 & 3 \\ 2 & 4 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -9+6 & 15+3 \\ -6+8 & 10+4 \\ 3-4 & -5-2 \end{bmatrix}$$

$$(3\times\underbrace{2)(2}_{=}\times 2) = 3\times 2$$

$$= \begin{bmatrix} -3 & 18 \\ 2 & 14 \\ -1 & -7 \end{bmatrix}$$

Find the product if it exists.

$$\begin{bmatrix} 3 & 3 \\ 2 & 4 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3+6 & -9+6 \\ 2+8 & -6+8 \\ -1-4 & 3-4 \end{bmatrix}$$

$(3\times2)(2\times2) = 3\times2$ ✓

$$= \begin{bmatrix} 9 & -3 \\ 10 & 2 \\ -5 & -1 \end{bmatrix}$$

Inverse Matrix:

- a 2 x 2 matrix will have an inverse if its determinant is not 0
- the product of a matrix and its inverse is the identity matrix

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse Matrix:

- Create an augmented matrix by "adding" the identity matrix onto the original matrix

$$\left[\begin{array}{cc:cc} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}\right]$$

- Row reduce the original matrix until it is the identity matrix

$$\left[\begin{array}{cc:cc} 1 & 0 & b_{13} & b_{14} \\ 0 & 1 & b_{23} & b_{24} \end{array}\right] \longleftarrow \text{this is the inverse matrix}$$

Find the inverse matrix, if it exists. Verify.

$$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$D = 15 - 14$

$D = 1$ ✓

$$\left[\begin{array}{cc:cc} 3 & 7 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{cccc} -2 & -\frac{14}{3} & -\frac{2}{3} & 0 \end{array} \\ \left[\begin{array}{cc:cc} 1 & \frac{7}{3} & \frac{1}{3} & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \end{array}$$

$$\rightarrow \left[\begin{array}{cc:cc} 1 & \frac{7}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right] \rightarrow \begin{array}{c} \left[\begin{array}{cc:cc} 1 & \frac{7}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -2 & 3 \end{array}\right] \\ \begin{array}{cccc} 0 & -\frac{7}{3} & \frac{14}{3} & -7 \end{array} \end{array}$$

$$\rightarrow \left[\begin{array}{cc:cc} 1 & 0 & 5 & -7 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}} \cdot \underset{A^{-1}}{\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}} = \begin{bmatrix} 15-14 & -21+21 \\ 10-10 & -14+15 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$(2\times2)(2\times2) = 2\times2 \checkmark$$

Find the inverse matrix, if it exists. Verify.

$$\begin{bmatrix} -3 & 4 \\ 4 & 5 \end{bmatrix}$$

$D = -15-16$

$D = -31 \checkmark$

$$\left[\begin{array}{cc|cc} -3 & 4 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{cccc} -4 & \frac{16}{3} & \frac{4}{3} & 0 \end{array} \\ \left[\begin{array}{cc|cc} 1 & -\frac{4}{3} & -\frac{1}{3} & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] \end{array} \rightarrow \left[\begin{array}{cc|cc} 1 & -\frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & \frac{31}{3} & \frac{4}{3} & 1 \end{array}\right]$$

$$\rightarrow \begin{array}{c} \left[\begin{array}{cc|cc} 1 & -\frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{4}{31} & \frac{3}{31} \end{array}\right] \\ \begin{array}{cccc} 0 & \frac{4}{3} & \frac{16}{93} & \frac{4}{31} \end{array} \end{array} \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -\frac{5}{31} & \frac{4}{31} \\ 0 & 1 & \frac{4}{31} & 0 \end{array}\right]$$

Assignment:

Inverse Matrices #1-8

Pick 2 inverses to verify

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