Today's Plan:

Learning Target (standard): I will solve a 2 x 2 linear system using the matrix method (row reduction).

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Cramer's Rule:

1) $D = 40$
$D_x = 100$
$D_y = -20$
$independent\left(\frac{5}{2}, -\frac{1}{2}\right)$

2) $D = 0$
$D_x = 7$
$inconsistent$
no solution

3) $D = -24$
$D_x = -60$
$D_y = -64$
$independent\left(\frac{5}{2}, \frac{8}{3}\right)$

4) $D = 14$
$D_x = -14$
$D_y = 56$
$D_z = 84$
$independent(-1, 4, 6)$

5) $D = 144$
$D_x = -234$
$D_y = 6$
$D_z = -162$
$independent\left(-\frac{13}{8}, \frac{1}{24}, -\frac{9}{8}\right)$

6) $D = 10$
$D_x = -130$
$D_y = -90$
$D_z = -100$
$independent(-13, -9, -10)$

Solve using Cramer's Rule.

$$-10y+10x=0 \qquad 10x-10y=0$$
$$2-4y=-6x \qquad 6x-4y=-2$$

$$D=\begin{vmatrix} 10 & -10 \\ 6 & -4 \end{vmatrix}=-40+60$$
$$D=20$$

$$D_x=\begin{vmatrix} 0 & -10 \\ -2 & -4 \end{vmatrix}=0-20$$
$$D_x=-20$$

$$D_y=\begin{vmatrix} 10 & 0 \\ 6 & -2 \end{vmatrix}=-20-0$$
$$D_y=-20$$

independent
$(-1,-1)$

Solve using Cramer's Rule.

$$3x-y+z=11$$
$$x+4y-2z=-12$$
$$2x+2y-z=-3$$

$$D=\begin{vmatrix} 3 & -1 & 1 \\ 1 & 4 & -2 \\ 2 & 2 & -1 \end{vmatrix}=3\begin{vmatrix} 4 & -2 \\ 2 & -1 \end{vmatrix}+\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}+\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix}$$
$$=3(-4+4)+(-1+4)+(2-8)$$
$$=0+3-6$$
$$D=-3$$

$$D_x=\begin{vmatrix} 11 & -1 & 1 \\ -12 & 4 & -2 \\ -3 & 2 & -1 \end{vmatrix}=11\begin{vmatrix} 4 & -2 \\ 2 & -1 \end{vmatrix}+\begin{vmatrix} -12 & -2 \\ -3 & -1 \end{vmatrix}+\begin{vmatrix} -12 & 4 \\ -3 & 2 \end{vmatrix}$$
$$=11(-4+4)+(12-6)+(-24+12)$$
$$=0+6-12 \qquad D_x=-6$$

$$D_y=\begin{vmatrix} 3 & 11 & 1 \\ 1 & -12 & -2 \\ 2 & -3 & -1 \end{vmatrix}=3\begin{vmatrix} -12 & -2 \\ -3 & -1 \end{vmatrix}-11\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}+\begin{vmatrix} 1 & -12 \\ 2 & -3 \end{vmatrix}$$
$$=3(12-6)-11(-1+4)+(-3+24)$$
$$=18-33+21 \qquad D_y=6$$

$$D_z=\begin{vmatrix} 3 & -1 & 11 \\ 1 & 4 & -12 \\ 2 & 2 & -3 \end{vmatrix}=3\begin{vmatrix} 4 & -12 \\ 2 & -3 \end{vmatrix}+\begin{vmatrix} 1 & -12 \\ 2 & -3 \end{vmatrix}+11\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix}$$
$$=3(-12+24)+(-3+24)+11(2-8)$$
$$=36+21-66 \qquad D_z=-9$$

independent
$(2,-2,3)$

The Matrix Method:

- the system of equations is solved by writing the system in matrix form and then performing row operations on the matrix
- uses an augmented matrix

Row Operations:

- Interchange any two rows
- Multiply all the elements in a row by the same nonzero number
- Replace a row by the sum of it and another row

$$3(x+2y=4)$$
$$3x-6y=10$$

$$3x+6y=12$$
$$3x-6y=10$$
$$\overline{\quad 6y=22 \quad}$$

Matrix Method on 2 x 2 systems:

$$a_1x+b_1y=c_1$$
$$=$$
$$a_2x+b_2y=c_2$$

$$\begin{array}{cccc} x & y & & = \\ \multicolumn{4}{c}{\left[\begin{array}{cc|c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}\right]} \end{array} \begin{array}{l} \leftarrow \text{1st equation} \\ \leftarrow \text{2nd equation} \end{array}$$

Process:

- change a_1 to a "1" - use operation 1 or 2
- change a_2 to a "0" - use operation 3
- change b_2 to a "1" - use operation 2

$$\left[\begin{array}{cc|c} 1 & \sim & \sim \\ 0 & 1 & \sim \end{array}\right]$$

Types of Solutions:

- Independent – last row $\begin{bmatrix} 0 & 1 & \vdots & \# \end{bmatrix}$ (x,y)
- Inconsistent – last row $\begin{bmatrix} 0 & 0 & \vdots & \# \end{bmatrix}$ no solution
- Dependent – last row $\begin{bmatrix} 0 & 0 & \vdots & 0 \end{bmatrix}$ infinite solutions

Solve the system using the matrix method: "Echelon form" "row reduction"

$$2x+5y=8$$

$$3x+4y=5$$

$$\left[\begin{array}{cc|c} 2 & 5 & 8 \\ 3 & 4 & 5 \end{array}\right] \xrightarrow{\div \text{ row 1 by 2}} \left[\begin{array}{cc|c} 1 & \frac{5}{2} & 4 \\ 3 & 4 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & \frac{5}{2} & 4 \\ & & \end{array}\right]$$