Today's Plan:

Learning Target (standard): I will solve a 3 x 3 linear system using the matrix method (row reduction).

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Matrix Method (2x2):

$1) independent (0, -5)$

$2) inconsistent$ no solution

$3) indepedent (8, -4)$

$4) independent (-1, 1)$

$5) independent (-10, 1)$

$6) independent (8, -8)$

Solve using Cramer's Rule.

$$6x - 6y = 5$$
$$2x - 10y = -1$$

$$D = \begin{vmatrix} 6 & -6 \\ 2 & -10 \end{vmatrix} = -60 + 12$$
$$\boxed{D = -48}$$

$$D_x = \begin{vmatrix} 5 & -6 \\ -1 & -10 \end{vmatrix} = -50 - 6$$
$$\boxed{D_x = -56}$$

independent
$$\left(\frac{7}{6}, \frac{1}{3}\right)$$

$$D_y = \begin{vmatrix} 6 & 5 \\ 2 & -1 \end{vmatrix} = -6 - 10$$
$$\boxed{D_y = -16}$$

Solve using the matrix method.

$$2x + y = -3$$
$$3x - 2y = -8$$

$$\begin{bmatrix} 2 & 1 & \vdots & -3 \\ 3 & -2 & \vdots & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 3 & -2 & \vdots & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & -\frac{7}{2} & \vdots & -\frac{7}{2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$x + \frac{1}{2}y = -\frac{3}{2}$$
$$y = 1$$

$$x + \frac{1}{2} = -\frac{3}{2}$$
$$x = -2$$

independent
$$(-2, 1)$$

Solve using the matrix method.

$$3x + y = 5$$

$$2x + 3y = 8$$

$$\begin{bmatrix} 3 & 1 & \vdots & 5 \\ 2 & 3 & \vdots & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{3} & \vdots & \frac{5}{3} \\ 2 & 3 & \vdots & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{3} & \vdots & \frac{5}{3} \\ 0 & \frac{7}{3} & \vdots & \frac{14}{3} \end{bmatrix}$$

$-2 \quad -\frac{2}{3} \quad -\frac{10}{3}$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{3} & \vdots & \frac{5}{3} \\ 0 & 1 & \vdots & 2 \end{bmatrix}$$

$$x + \frac{1}{3}y = \frac{5}{3} \qquad x + \frac{2}{3} = \frac{5}{3}$$

$$y = 2 \qquad\qquad x = 1$$

independent
(1,2)

Solve using the matrix method.

$$8x - 2y = 20$$

$$5x + 2y = -7$$

$-5 \quad \frac{5}{4} \quad -\frac{25}{2}$

$$\begin{bmatrix} 8 & -2 & \vdots & 20 \\ 5 & 2 & \vdots & -7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & \vdots & \frac{5}{2} \\ 5 & 2 & \vdots & -7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & \vdots & \frac{5}{2} \\ 0 & \frac{13}{4} & \vdots & -\frac{39}{2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & \vdots & \frac{5}{2} \\ 0 & 1 & \vdots & -6 \end{bmatrix}$$

$$x - \frac{1}{4}y = \frac{5}{2} \qquad x + \frac{3}{2} = \frac{5}{2}$$

$$y = -6 \qquad\qquad x = 1$$

independent
(1,-6)

Solve using the matrix method.

$$7x + 7y = -7$$

$$-4x - 2y = -16$$

4   4   -4

$$\begin{bmatrix} 7 & 7 & \vdots & -7 \\ -4 & -2 & \vdots & -16 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & \vdots & -1 \\ -4 & -2 & \vdots & -16 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & \vdots & -1 \\ 0 & 2 & \vdots & -20 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & \vdots & -1 \\ 0 & 1 & \vdots & -10 \end{bmatrix}$$

$x + y = -1$ $x - 10 = -1$

$y = -10$ $x = 9$

independent

$(9, -10)$

Matrix Method on 3 x 3 Systems:

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & \vdots & d_1 \\ a_2 & b_2 & c_2 & \vdots & d_2 \\ a_3 & b_3 & c_3 & \vdots & d_3 \end{bmatrix}$$

Process:
- change a_1 to a "1" - operation 1 or 2
- change a_2 to a "0" - operation 3
- change a_3 to a "0" - operation 3 on row 1
- change b_2 to a "1" - operation 1 or 2
- ✱ change b_3 to a "0" - operation 3 on row 2
- change c_3 to a "1" - operation 2

$$\begin{bmatrix} 1 & \sim & \sim & \vdots & \sim \\ 0 & 1 & \sim & \vdots & \sim \\ 0 & 0 & 1 & \vdots & \sim \end{bmatrix}$$

Types of Solutions:

- Independent — last row $\begin{bmatrix} 0 & 0 & 1 & \vdots & \# \end{bmatrix}$
  (x, y, z)

- Inconsistent — last row $\begin{bmatrix} 0 & 0 & 0 & \vdots & \# \end{bmatrix}$
  no solution

- Dependent — last row $\begin{bmatrix} 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$
  infinite
     solutions

Use the matrix method to solve each system:

$2x + 3y + 3z = -2$
$x + 2y - 3z = 9$
$3x - 2y - 4z = 1$

-2 -4 6 -18   -3 -6 9 -27

$\begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 2 & 3 & 3 & \vdots & -2 \\ 3 & -2 & -4 & \vdots & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 0 & -1 & 9 & \vdots & -20 \\ 3 & -2 & -4 & \vdots & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 0 & -1 & 9 & \vdots & -20 \\ 0 & -8 & 5 & \vdots & -26 \end{bmatrix}$

0 8 -72 160

$\rightarrow \begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 0 & 1 & -9 & \vdots & 20 \\ 0 & -8 & 5 & \vdots & -26 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 0 & 1 & -9 & \vdots & 20 \\ 0 & 0 & -67 & \vdots & 134 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 2 & -3 & \vdots & 9 \\ 0 & 1 & -9 & \vdots & 20 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$

independent
$(-1, 2, -2)$

$x + 2y - 3z = 9$
$y - 9z = 20$
$z = -2$

$y + 18 = 20$
$y = 2$

$x + 4 + 6 = 9$
$x + 10 = 9$
$x = -1$

Assignment:

- Matrix Method 2x2
    More Practice #1-6