Today's Plan:

Learning Target (standard): I will find the "c" guaranteed by the mean value theorem. I will explain the meaning of this "c."

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

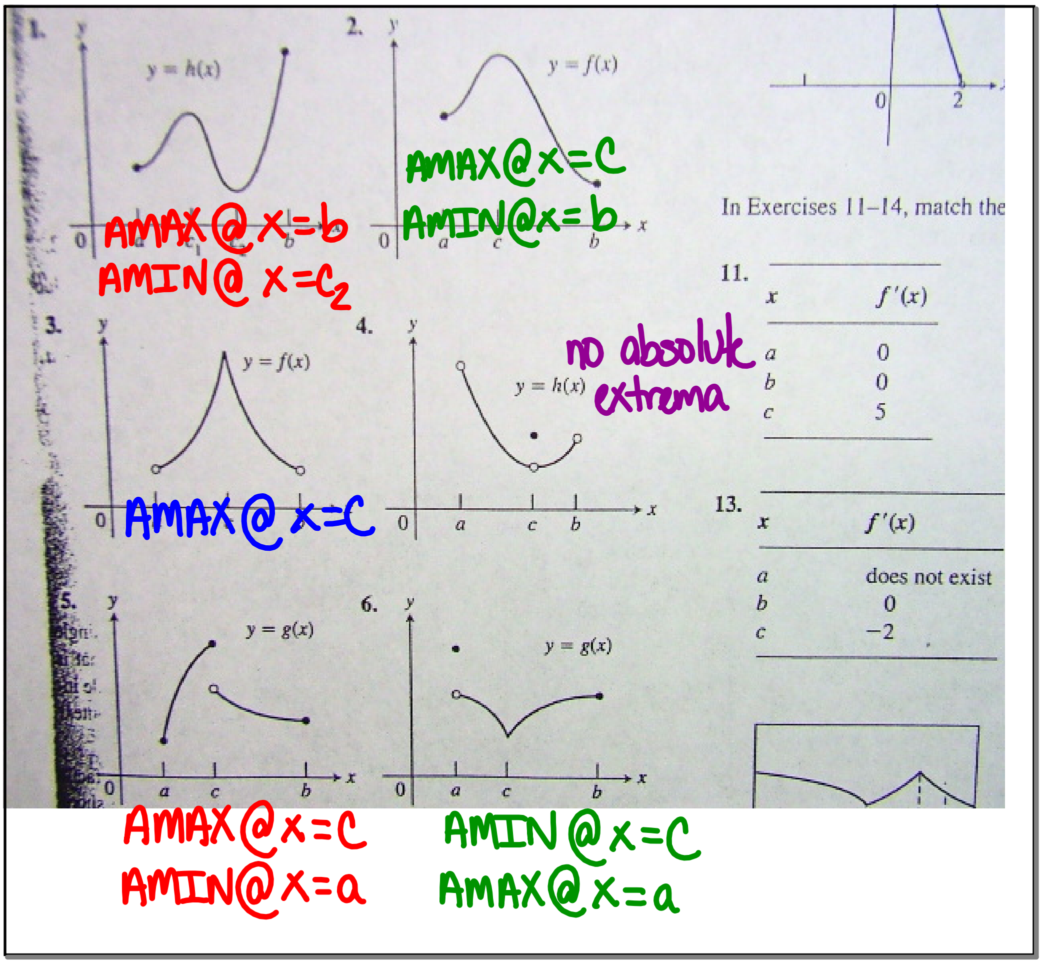
Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Assignment: "Exercises 3.1" #1-14

- #1-6 identify the location of the AMAX & AMIN
- #7-10 identify the location of ALL extreme values
  - both local and absolute extrema
- #11-14 match the tables to the graphs



1. $y = h(x)$

AMAX@x=b
AMIN@ x=c_2

2. $y = f(x)$

AMAX@x=c
AMIN@x=b

3. $y = f(x)$

AMAX@x=c

4. $y = h(x)$

no absolute extrema

5. $y = g(x)$

AMAX@x=c
AMIN@x=a

6. $y = g(x)$

AMIN@x=c
AMAX@x=a

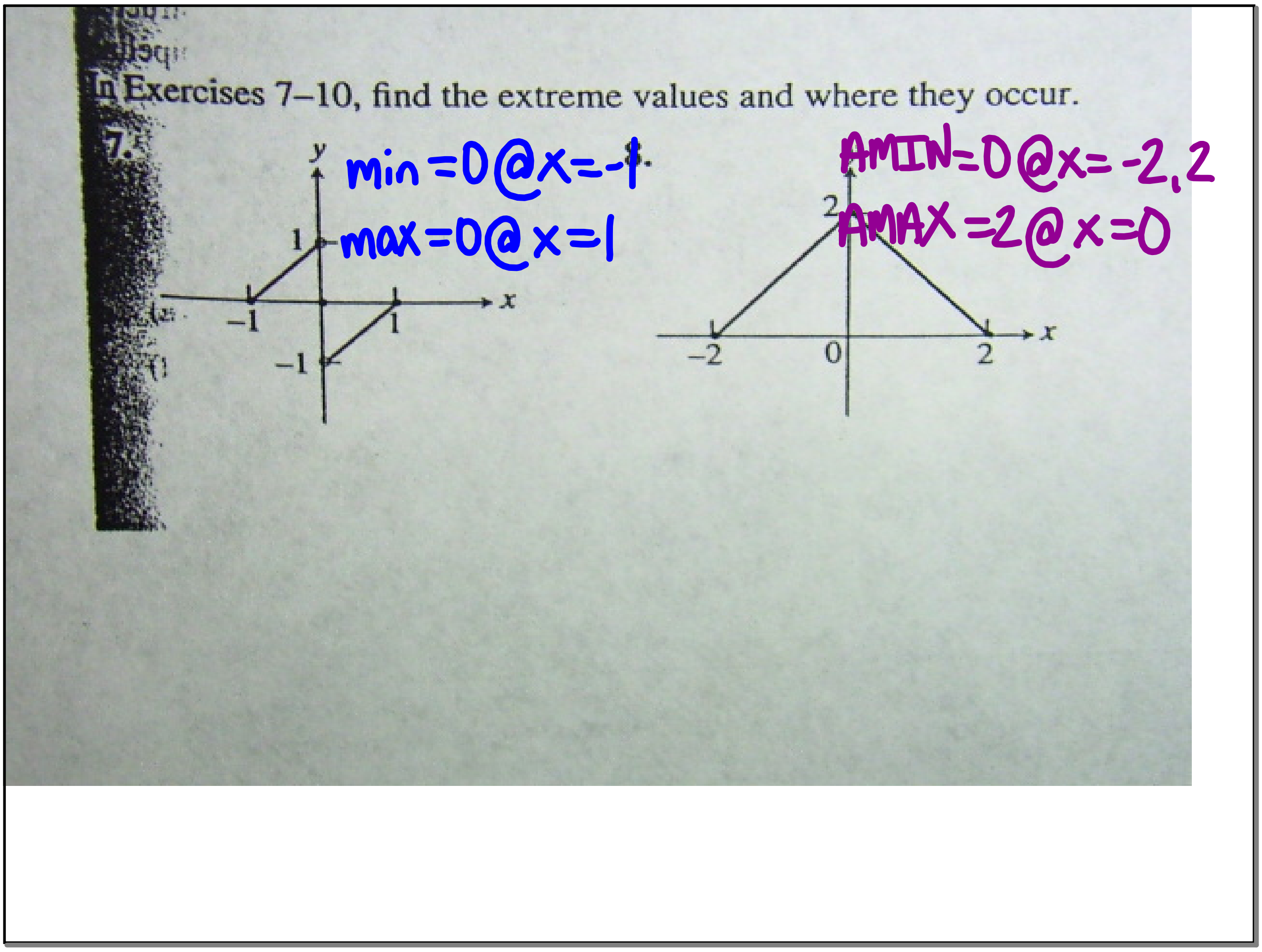
In Exercises 11–14, match the

11.

x	$f'(x)$
a	0
b	0
c	5

13.

x	$f'(x)$
a	does not exist
b	0
c	-2



In Exercises 7–10, find the extreme values and where they occur.

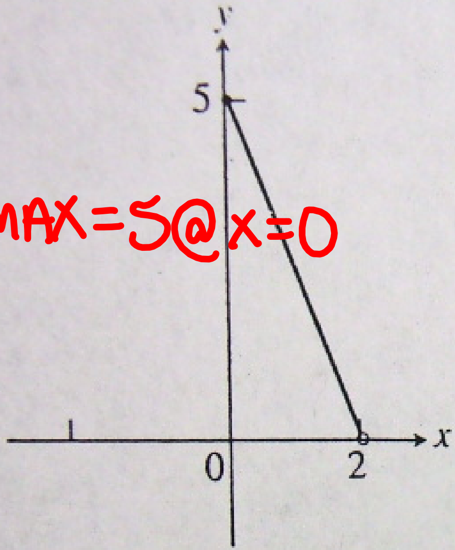
7.

min=0@x=-1
max=0@x=1

AMIN=0@x=-2,2
AMAX=2@x=0

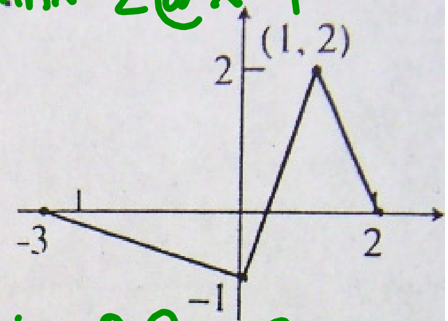
9.

AMAX=5@x=0



AMIN=-1@x=0

AMAX=2@x=1



min=0@x=2

max=0@x=-3

In Exercises 11–14, match the table with a graph.

11.

x	$f'(x)$
a	0
b	0
c	5

c

12.

x	$f'(x)$
a	0
b	0
c	−5

b

13.

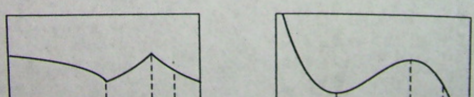
x	$f'(x)$
a	does not exi
b	0
c	−2

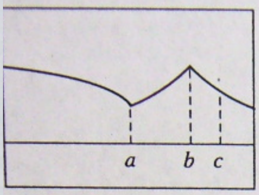
d

14.

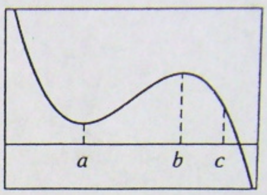
x	$f'(x)$
a	does not exist
b	does not exist
c	−1.7

a

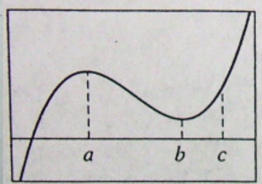




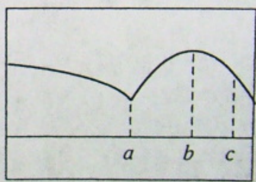
(a)



(b)



(c)



(d)

Find the absolute extremum.

$f(x) = x^3 - 2x^2 + x; [0,4]$

$f'(x) = 3x^2 - 4x + 1$

$0 = (3x-1)(x-1)$

Critical #s:

$x = \frac{1}{3}, 1$

$f(\frac{1}{3}) = (\frac{1}{3})^3 - 2(\frac{1}{3})^2 + \frac{1}{3}$

$= \frac{1}{27} - \frac{2}{9} +$

$= \frac{1}{27} - \frac{6}{27} + \frac{9}{27}$

$f(\frac{1}{3}) = \frac{4}{27}$

$f(0) = 0^3 - 2(0) + 0$

$f(0) = 0$

$f(4) = 4^3 - 2(4)^2 + 4$

$= 64 - 32 + 4$

$f(4) = 36$

$f(1) = 1^3 - 2(1)^2 + 1$

$= 1 - 2 + 1$

$f(1) = 0$

$\therefore$ AMAX $= 36$ @ $x = 4$

AMIN $= 0$ @ $x = 0, 1$

Find the absolute extremum.

$f(x) = 2x^3 + x^2 - 1; [1,4]$

$f'(x) = 6x^2 + 2x$

$0 = 2x(3x+1)$

Critical #s:

$x = 0, -\frac{1}{3}$

$x = 0, -\frac{1}{3}$ are not in $[1,4]$

$f(1) = 2(1)^3 + (1)^2 - 1$

$= 2 + 1 - 1$

$f(1) = 2$

$f(4) = 2(4)^3 + 4^2 - 1$

$= 128 + 16 - 1$

$f(4) = 143$

$\therefore$ AMIN $= 2$ @ $x = 1$

AMAX $= 143$ @ $x = 4$

The Mean Value Theorem:

https://www.khanacademy.org/math/ap-calculus-ab/ab-diff-analytical-applications-new/ab-5-1/v/mean-value-theorem-1

- an application of differentiation and continuity
- physical and graphical interpretations

The Mean Value Theorem

If f(x) is continuous on [a,b] and differentiable on (a,b), then there is a number x = c between a and b for which

$$f(b) - f(a) = (f'(c))(b - a).$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$m_{tangent}$
"instantaneous" rate of change

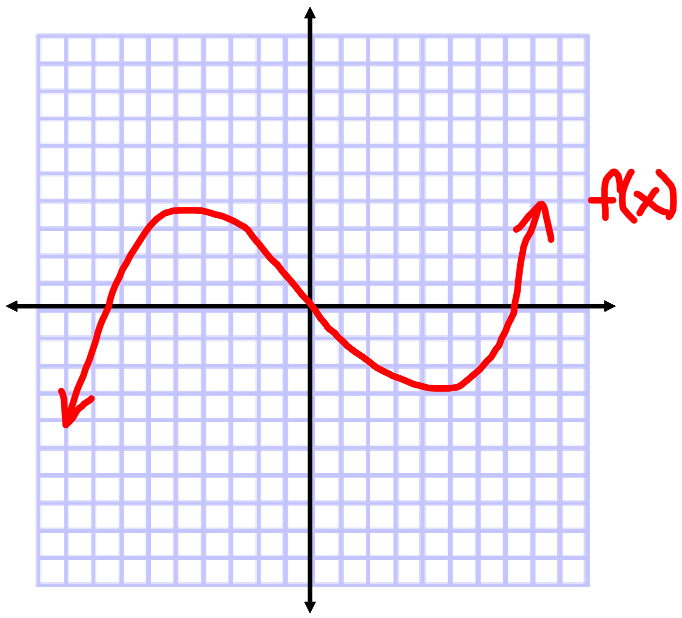
m_{secant}
AROC
"average" rate of change
"mean"

tangent // secant
$m_{tangent} = m_{secant}$

Interpretation 1

Graphical --if a function is continuous on [a,b] and differentiable on (a,b), then there is a point x = c on (a,b) such that the tangent line through c will be parallel to the secant line through a and b.

$a < c < b$



Interpretation 2

Physical –[f(b) –f(a)]/(b –a) is the average rate of change in f(x) over [a,b] and f ’ (*c*) is the instantaneous change.

The MVT says that the instantaneous change at some interior point must equal the average change over the entire interval.

Examples

If a car travels 200 miles in 4 hours, its average velocity is 50 mph. At some point during that 4 hours, the speedometer must read exactly 50 mph.

Determine whether $f(x) = x^3 - 8x - 5$ satisfies the hypothesis of the MVT on [1,4] and find c that satisfies the conclusion. $[a,b]$

Continuous $[1,4]$ ✓

$f'(x) = 3x^2 - 8$ differentiable $(1,4)$ ✓

$f(b) - f(a) = (b-a)f'(c)$

$f(4) - f(1) = (4-1)(3c^2 - 8)$

$27 + 12 = 3(3c^2 - 8)$

$39 = 3(3c^2 - 8)$

$13 = 3c^2 - 8$

$21 = 3c^2$

$\pm\sqrt{7} = \sqrt{c^2}$

$c = \sqrt{7}, -\sqrt{7}$ not in $[1,4]$

$c = \sqrt{7}$

Assignment:

- p.144 #9-11,13,14