Today's Plan:

Learning Target (standard): I will evaluate summations.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Series #1-10

1)358
2)168
3)413
4)85
5)164
6)470
7)15
8)2945
9)279
10)20

Find the sum of the series.

$$\sum_{k=1}^{9} -(-2)^k = [-(-2)^1]+[-(-2)^2]+[-(-2)^3]+[-(-2)^4]$$
$$+[-(-2)^5]+[-(-2)^6]+[-(-2)^7]+[-(-2)^8]+[-(-2)^9]$$
$$=2-4+8-16+32-64+128-256+512$$
$$=342$$

Find the sum of the series.

$$\sum_{k=3}^{6} (2^k-1) = (2^3-1)+(2^4-1)+(2^5-1)+(2^6-1)$$
$$=7+15+31+63$$
$$=116$$

Find the sum of the series.

$$\sum_{k=2}^{7}(3k+4) = [3(2)+4] + [3(3)+4] + [3(4)+4] + [3(5)+4] + [3(6)+4] + [3(7)+4]$$

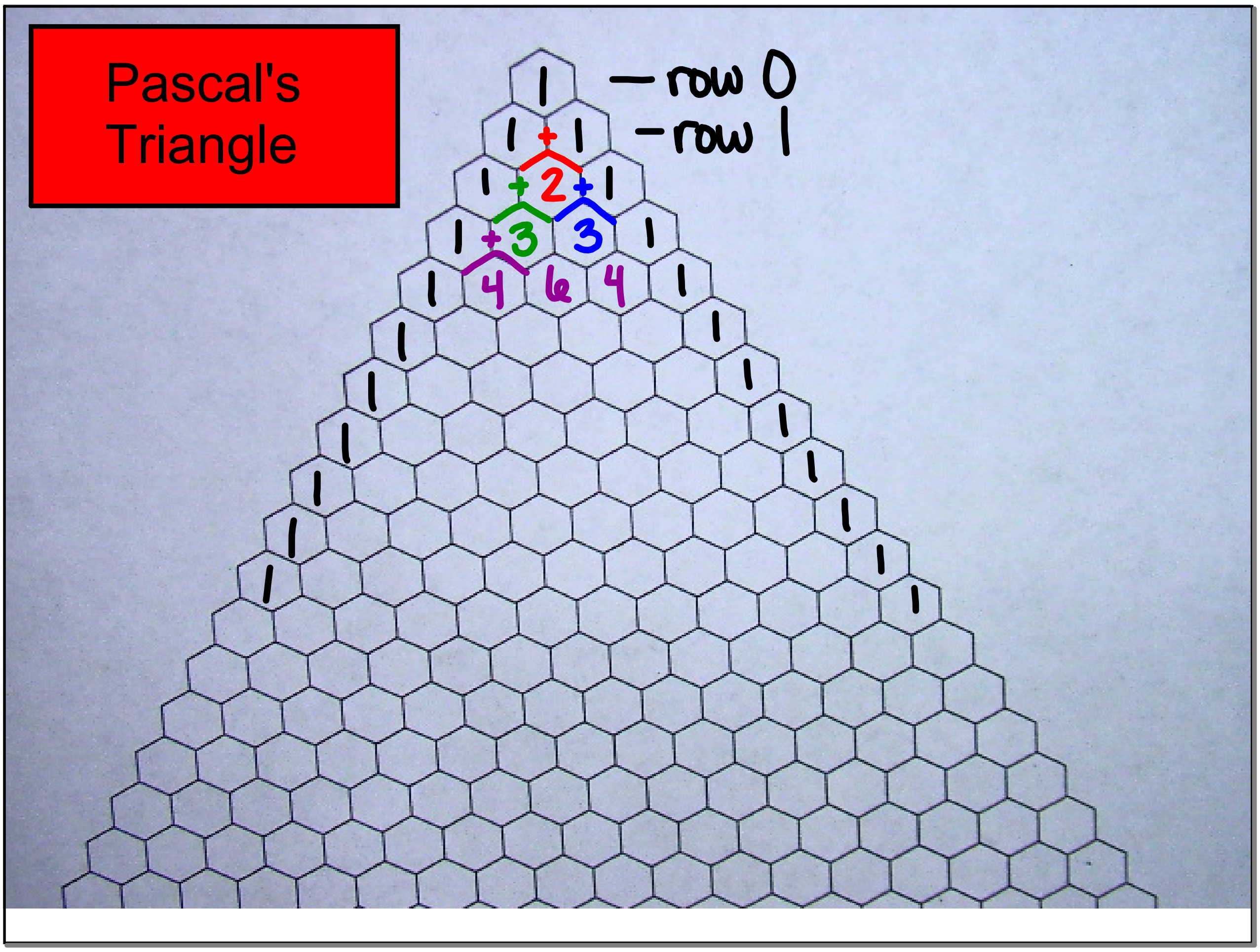
$$= 10 + 13 + 16 + 19 + 22 + 25$$

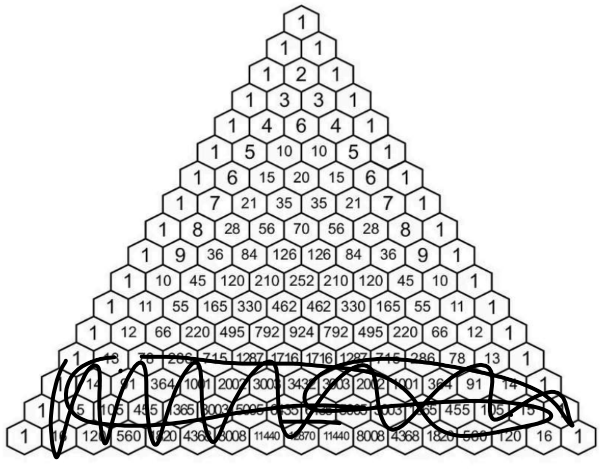
$$= 105$$

Pascal's Triangle is arguably the most famous triangle, ever. Despite being named after 17th century French mathematician Blaise Pascal, it was actually discovered by Chinese mathematician Jia Xian in the 11th century.

Taught to millions of high schoolers worldwide as a representation of the coefficients in a binomial expansion, but did you know that **it also has a few lesser known properties that lie just under the surface?**

$$(2x-3y)^7$$





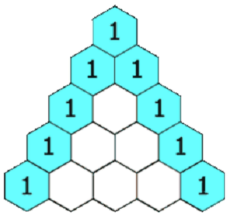
via Desmos

1) Sums

This is the most basic property of Pascal's triangle

> Every number is equal to the sum of the 2 numbers above it.

This is the easiest way in which Pascal's triangle can be generated — repeatedly adding the cells in a layer in order to generate the layer below.



via Geo-Numerology

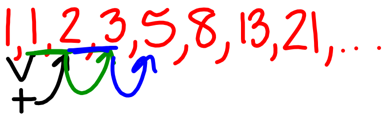
> *This is fine for the first few layers, but it can get extremely tedious very quickly as the row length and the numbers in the row both increase.*

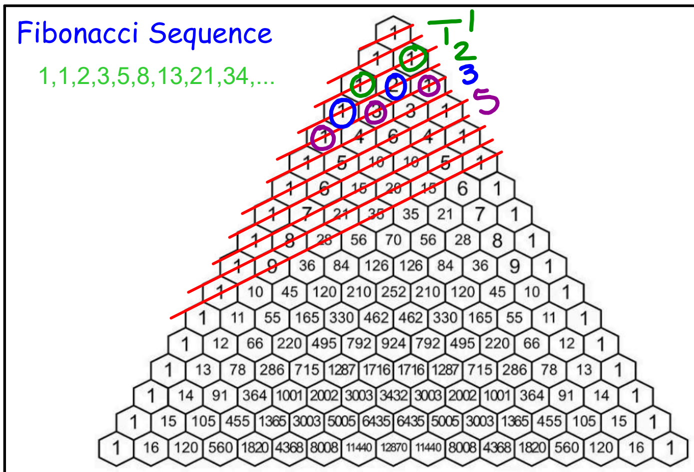
2) Fibonacci Series

Another secret of Pascal's Triangle is the presence of the Fibonacci series, the series where each number is equal to the sum of the two preceding value.

To find this series, you have to draw "**shallow diagonals**" through the triangle, starting from the top and moving bottom.

> The sum of these shallow diagonals are the terms in the Fibonacci series, starting with 1 and continuing down the triangle.





via Desmos

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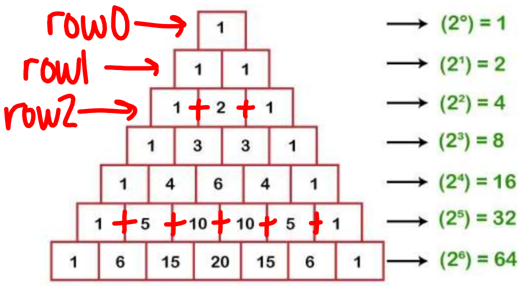
This is the easiest way in which Pascal's triangle can be generated — repeatedly adding the cells in a layer in order to generate the layer below.

The sum of the numbers in the nth row of Pascal's triangle is equal to 2^n (starting with row 0).



via JavaTPoint

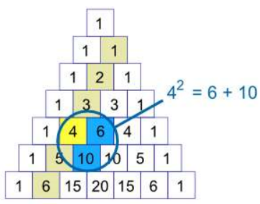
For example, the sum of the numbers in the first row is 1, or 2^0. The second row is $2 = 2^1$, the third row is $4 = 2^2$, and so on.

4) Number Sequences

Pascal's Triangle can be broken up into columns which start at any term in the right edge and continue through the terms to the direct bottom right to it. Each column has values of a different number sequence.

The columns of Pascal's triangle have values of a different numerical series in which the common

If you take any number in the second column (the counting numbers), the square of the number is equal to the sum of the numbers on its right and bottom right.



via USU

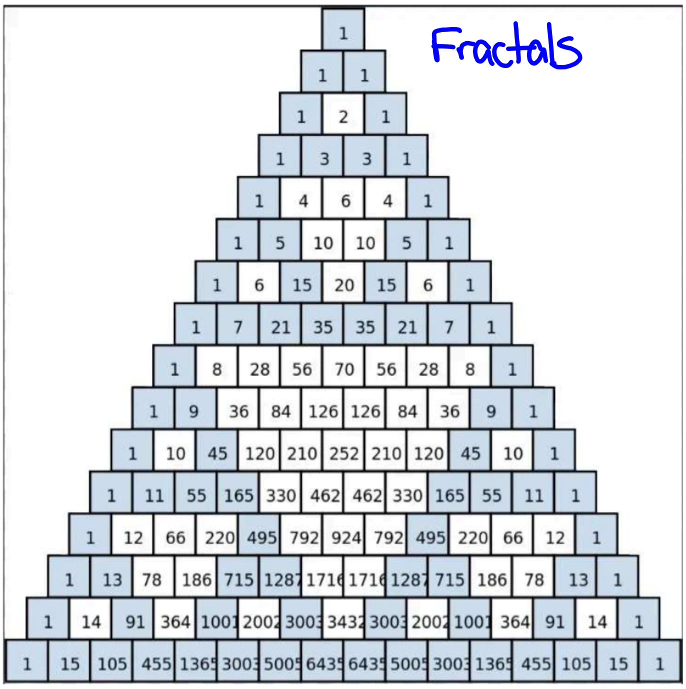
Take the example of the number 4 in the 5th row. `4 * 4 = 16`, which is equal to 6 (the number to its right) plus 10 (the number to its bottom-right). This works for any number in the second column.

6) Sierpinski Triangle

The Sierpinski Triangle is definitely one of my favourite properties of Pascal's triangle. A fractal is a shape that is infinitely repeated, no matter how small/large it is.

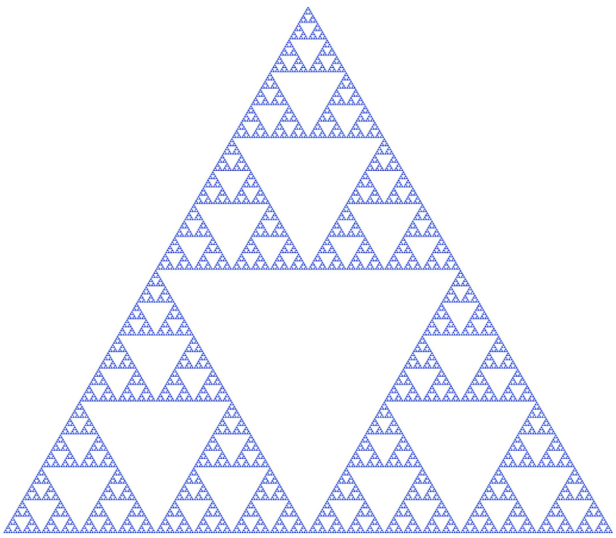
If you zoom into a fractal, you will discover a smaller version of the original image, which continues forever. This means that fractals can never be physically constructed, as every physical object can only have changes larger than a single atom.

Fractals



via FractalFoundation

Suppose every odd number is shaded in. On the triangle above, you might just about be able to see a fractal — every triangle is divided into 4 sub-triangles. *This fractal is known as a Sierpinski triangle.*



via Wikipedia

7) Binomial Expansion

This is probably the use that most people are familiar with. The rows in Pascal's triangle are the values of the coefficients in a binomial expansion.

Binomial Expansion Theorem:

$$(ax+by)^n = \binom{\#n-row}{1^{st}\,spot}(ax)^n(by)^0 + \binom{\#n-row}{2^{nd}\,spot}(ax)^{n-1}(by)^1 + \binom{\#n-row}{3^{rd}\,spot}(ax)^{n-2}(by)^2 + \cdots + \binom{\#n-row}{2^{nd}\,to-last}(ax)^1(by)^{n-1} + \binom{\#n-row}{last-spot}(ax)^0(by)^n$$

$$(ax-by)^n = (ax+(-by))^n$$

Assignment:

Series Practice #1-10