

Today's Plan:

Learning Target (standard): I will use the remainder theorem and factor to theorem to locate zeros of polynomials. I will use the zeros to factor the polynomial.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Real Zeros of Polynomials

1) $no - f(2) = -6$

2) $no - f(-3) = 7$

3) $q(x) = x^2 + 6x - 6 - \frac{9}{x-2}$

4) $q(n) = n^3 + 5n^2 + 5n - 1 + \frac{3}{n+3}$

5) $f(x) = (x+3)(3x-1)(x+1)$

zeros : $x = -3, \frac{1}{3}, 1$

QUIZ Monday

Real Zeros of Polynomials

$$6) f(x) = (x-5)(x+i)(x-i)(\sqrt{3}x+\sqrt{2})(\sqrt{3}x-\sqrt{2})$$

$$\text{zeros : } x = 5, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}, i, -i$$

$$7) f(x) = (x-2)(\sqrt{2}x+i)(\sqrt{2}x-i)(x+3i)(x-3i)$$

$$\text{zeros : } x = 2, \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2}i, -3i, 3i$$

$$8) f(x) = 3(3x+5)(x-4-\sqrt{23})(x-4+\sqrt{23})$$

$$\text{zeros : } x = -\frac{5}{3}, 4+\sqrt{23}, 4-\sqrt{23}$$

Real Zeros of Polynomials

$$9) f(x) = 3(3x+1)(\sqrt{2}x+\sqrt{3})(\sqrt{2}x-\sqrt{3})(x+\sqrt{2}i)(x-\sqrt{2}i)$$

$$\text{zeros : } x = -\frac{1}{3}, \frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}, \sqrt{2}i, -\sqrt{2}i$$

$$10) f(x) = (x+5)(\sqrt{3}x+1)(\sqrt{3}x-1)(x+\sqrt{5})(x-\sqrt{5})$$

$$\text{zeros : } x = -5, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \sqrt{5}, -\sqrt{5}$$

5) $f(x) = (x+3)(x+1)(3x-1)$
 Zeros: $x = -3, -1, \frac{1}{3}$

6) $f(x) = (x-5)(3x^2-2)(x^2+1)$ $x^2+1=0$
 $x^2=-1$
 $x=i, -i$

$f(x) = (x-5)(\sqrt{3}x+\sqrt{2})(\sqrt{3}x-\sqrt{2})(x+i)(x-i)$
 Zeros: $x = 5, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}, i, -i$

$3x^2-2=0$
 $3x^2=2$
 $x^2 = \frac{2}{3}$
 $x = \frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}}$
 $x = \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}$

7) $f(x) = 2x^5 - 4x^4 + 19x^3 - 38x^2 + 9x - 18$
 MNZ: 5
 P: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
 Q: $\pm 1, \pm 2$
 $\frac{P}{Q}$: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6, \pm 9, \pm \frac{9}{2}, \pm 18$

2	2	-4	19	-38	9	-18
	4	0	38	0	18	
	2	0	19	0	9	0

$f(x) = (x-2)(2x^4+19x+9)$ $2x^2+1=0$
 $2x^2=-1$
 $x^2=-\frac{1}{2}$

$f(x) = (x-2)(2x^2+1)(x^2+9)$

$f(x) = (x-2)(\sqrt{2}x+i)(\sqrt{2}x-i)(x+3i)(x-3i)$
 Zeros: $x = 2, -\frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2}i, -3i, 3i$

$$8) f(x) = (3x+5)(x^2-8x-7) \quad x=4+\sqrt{23}, 4-\sqrt{23}$$

$$f(x) = (3x+5)(x-4+\sqrt{23})(x-4-\sqrt{23})$$

Zeros: $x = -\frac{5}{3}, 4+\sqrt{23}, 4-\sqrt{23}$

$$x = -\frac{5}{3}$$

$$3x = -5$$

$$3x+5=0$$

$$9) f(x) = 6x^5 + 2x^4 + 3x^3 + x^2 - 18x - 6$$

MNZ: 5

P: $\pm 1, \pm 2, \pm 3, \pm 6$

Q: $\pm 1, \pm 2, \pm 3, \pm 6$

$\frac{P}{Q}$: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm \frac{3}{2}, \pm 6$

$$\begin{array}{r|rrrrrr} -\frac{1}{3} & 6 & 2 & 3 & 1 & -18 & -6 \\ & & -2 & 0 & -1 & 0 & 6 \\ \hline & 6 & 0 & 3 & 0 & -18 & 0 \end{array}$$

$$f(x) = (3x+1)(6x^4+3x^2-18)$$

$$f(x) = 3(3x+1)(2x^4+x^2-6)$$

$$\sqrt{2}x+\sqrt{3}=0$$

$$\sqrt{2}x=-\sqrt{3}$$

$$f(x) = 3(3x+1)(2x^2-3)(x^2+2) \quad x = \frac{-\sqrt{3}}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}$$

$$f(x) = 3(3x+1)(\sqrt{2}x+\sqrt{3})(\sqrt{2}x-\sqrt{3})(x+\sqrt{2})(x-\sqrt{2})$$

Zeros: $x = -\frac{1}{3}, -\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}, -\sqrt{2}, \sqrt{2}$

$$10) f(x) = 3x^5 + 15x^4 - 16x^3 - 80x^2 + 5x + 25$$

MNZ: 5

$$p: \pm 1, \pm 5, \pm 25$$

$$q: \pm 1, \pm 3$$

$$\frac{p}{q}: \pm 1, \pm \frac{1}{3}, \pm 5, \pm \frac{5}{3}, \pm 25, \pm \frac{25}{3}$$

$$\begin{array}{r|rrrrrr} -5 & 3 & 15 & -16 & -80 & 5 & 25 \\ & & -15 & 0 & 80 & 0 & -25 \\ \hline & 3 & 0 & -16 & 0 & 5 & 0 \end{array}$$

$$f(x) = (x+5)(3x^4 - 16x^2 + 5)$$

$$f(x) = (x+5)(3x^2 - 1)(x^2 - 5)$$

$$f(x) = (x+5)(\sqrt{3}x+1)(\sqrt{3}x-1)(x+\sqrt{5})(x-\sqrt{5})$$

Zeros: $x = -5, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\sqrt{5}, \sqrt{5}$

$$\sqrt{3}x+1=0$$

$$\sqrt{3}x=-1$$

$$x = -\frac{1}{\sqrt{3}}$$

$$x = -\frac{\sqrt{3}}{3}$$

$$x = \frac{\sqrt{3}}{3}$$

Simplify.

$$(4x^4 - 6x^2 + 3x - 1) \div (x - 2)$$

$$\begin{array}{r|rrrrr} 2 & 4 & 0 & -6 & 3 & -1 \\ & & 8 & 16 & 20 & 46 \\ \hline & 4 & 8 & 10 & 23 & 45 \end{array}$$

$$4x^3 + 8x^2 + 10x + 23 + \frac{45}{x-2}$$

Simplify.

$$\frac{3x^3}{3x} = x^2 \quad -\frac{x^2}{3x} = -\frac{1}{3}x$$

$$(3x^3 - 2x^2 + x - 1) \div (3x - 1)$$

$$\begin{array}{r}
 x^2 - \frac{1}{3}x + \frac{2}{9} \\
 \hline
 3x-1 \overline{) 3x^3 - 2x^2 + x - 1} \\
 \underline{-3x^3 + x^2} \\
 -x^2 + x - 1 \\
 + x^2 - \frac{1}{3}x \\
 \frac{2}{3}x - 1 \\
 \underline{-\frac{2}{3}x + \frac{2}{9}} \\
 \phantom{-\frac{2}{3}x} -\frac{7}{9}
 \end{array}$$

$$x^2 - \frac{1}{3}x + \frac{2}{9} + \frac{-7}{9(3x-1)}$$

$$\begin{array}{l}
 \frac{2}{3} \div 3 \\
 \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}
 \end{array}$$

List the potential rational zeros.

$$f(x) = \underline{-4}x^3 - x^2 + x + \underline{2}$$

$$MNZ: 3$$

$$p: \pm 1, \pm 2$$

$$q: \pm 1, \pm 2, \pm 4$$

$$\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2$$

List the potential rational zeros.

$$f(x) = \underline{3}x^4 - 3x^3 + x^2 - x + \underline{1}$$

$$MNZ: 4$$

$$p: \pm 1$$

$$q: \pm 1, \pm 3$$

$$\frac{p}{q}: \pm 1, \pm \frac{1}{3}$$

Find the real zeros & completely factor:

$$f(x) = 8x^3 - x^2 + 8x - 1$$

$$f(x) = x^2(8x-1) + 1(8x-1)$$

$$f(x) = (8x-1)(\underline{x^2+1})$$

$$\text{zeros: } x = \frac{1}{8}$$

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = i, -i$$

→ not real

Use the Factor Theorem to determine if the given binomial is a factor of the function.

$$f(x) = 4x^6 - 64x^4 + x^2 - 15; \underline{x+4}$$

$$\begin{aligned} f(-4) &= 4(-4)^6 - 64(-4)^4 + (-4)^2 - 15 \\ &= 16384 - 16384 + 16 - 15 \end{aligned}$$

$$f(-4) = 1$$

\therefore According to the Factor Theorem $x+4$ is not a factor of $f(x)$ because $f(-4) = 1$ and that is the remainder.

Factor completely.

$$216x^{6n} - y^{9n}$$

$$(6x^{2n} - y^{3n})(36x^{4n} + 6x^{2n}y^{3n} + y^{6n})$$

Solve using the quadratic formula.

$$6p^2 - 18 = 2p$$

$$6p^2 - 2p - 18 = 0$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(6)(-18)}}{2(6)}$$

$$= \frac{2 \pm \sqrt{4 + 432}}{12}$$

$$= \frac{2 \pm \sqrt{436}}{12}$$

~~$$= \frac{2 \pm 2\sqrt{109}}{12}$$~~

$$p = \frac{1 + \sqrt{109}}{6}, \frac{1 - \sqrt{109}}{6}$$

$$= \frac{2 \pm 2\sqrt{109}}{12}$$

$$p = \frac{1}{6} + \frac{\sqrt{109}}{6}, \frac{1}{6} - \frac{\sqrt{109}}{6}$$

Solve & completely factor:

$$x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 16 = 0$$

MN2: 5

$p: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$q: \pm 1$

$$\begin{array}{r|rrrrrr} 1 & 1 & -5 & 12 & -24 & 32 & -16 \\ & & 1 & -4 & 8 & -16 & 16 \\ \hline & 1 & -4 & 8 & -16 & 16 & 0 \end{array}$$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$(x-1)(x^4 - 4x^3 + 8x^2 - 16x + 16) = 0$$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$$(x-1)(x-2)(x^3 - 2x^2 + 4x - 8) = 0$$

$$x^2(x-2) + 4(x-2) = 0$$

$$(x-2)(x^2 + 4) = 0$$

$$(x-1)(x-2)^2(x+2i)(x-2i) = 0$$

Zeros: $x = 1, 2, -2i, 2i$

Assignment:

Quadratic & Factoring Review

#1-13

*** QUIZ Monday ***

Quadratics & Factoring Review:

$$1) x = -7, -6$$

$$2) n = 5, 8$$

$$3) r = \sqrt{10}, -\sqrt{10}$$

$$4) x = 2\sqrt{2}i, -2\sqrt{2}i$$

$$5) k = -5, 1$$

$$6) x = -\frac{3}{2}, 11$$

$$7) a = \frac{2}{9} + \frac{5\sqrt{2}}{9}i, \frac{2}{9} - \frac{5\sqrt{2}}{9}i$$

$$8) b = \frac{4}{7} + \frac{\sqrt{5}}{7}i, \frac{4}{7} - \frac{\sqrt{5}}{7}i$$

$$9) -3(u+3)(u^2 - 3u + 9)$$

$$10) 5(a+5)(9a-8)$$

$$11) f(-3) = 2$$

$$12) f(6) = -12$$

$$13) f(x) = (x-3)(x+i)(x-i)(\sqrt{2}x + \sqrt{5}i)(\sqrt{2}x - \sqrt{5}i)$$

$$\text{zeros : } x = 3, -i, i, -\frac{\sqrt{10}}{2}i, \frac{\sqrt{10}}{2}i$$