Today's Plan:

Learning Target (standard): I will graph rational functions using the 7-step process.

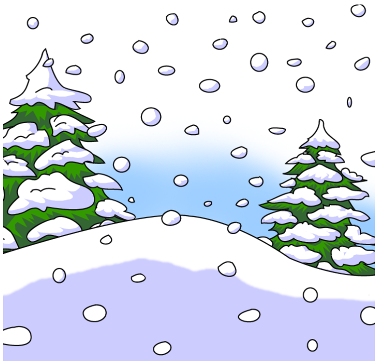
Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

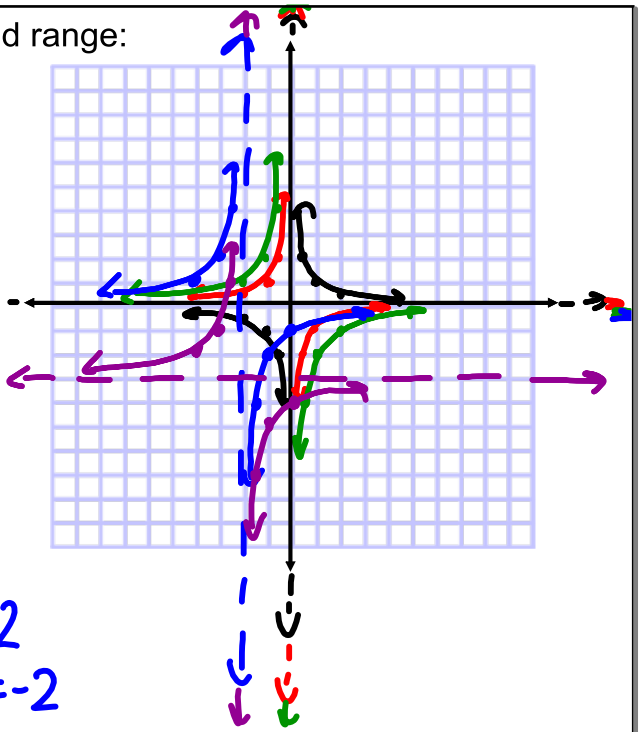
Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Please take a few minutes to go over your graphs with someone near you!



Graph and find the domain and range:

$$f(x)=\frac{-8}{4x+8}-3$$



$f(x)=\frac{-2}{x+2}-3$

parent: $f(x)=\frac{1}{x}$ VA: $x=0$ HA: $y=0$

1) $f(x)=\frac{-1}{x}$ r_x

2) $f(x)=-\frac{2}{x}$ v.s. by 2

3) $f(x)=\frac{-2}{x+2}$ shift left 2

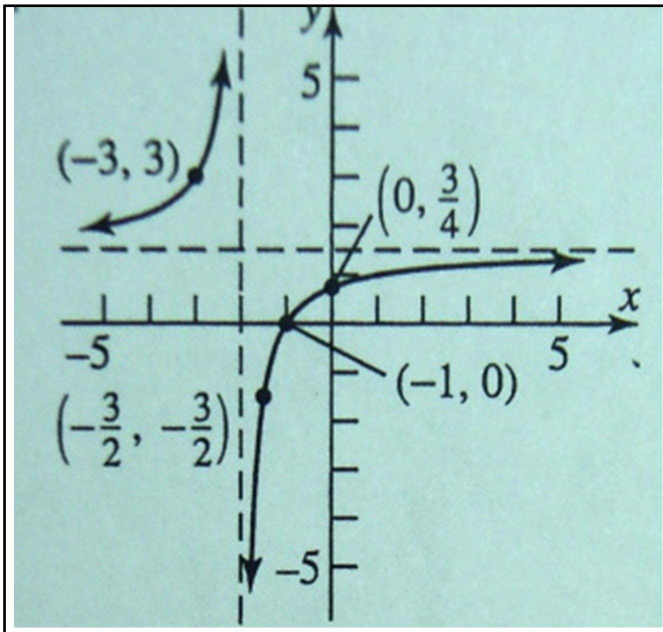
VA: $x=-2$

4) $f(x)=\frac{-2}{x+2}-3$ shift down 3

HA: $y=-3$

D: $\{x \mid x\neq -2\}$

R: $\{y \mid y\neq -3\}$



D: $\{x \mid x\neq -2\}$

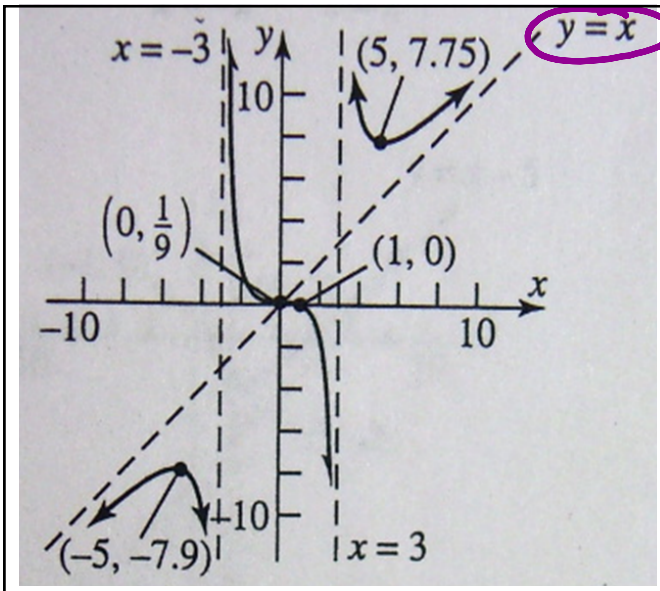
R: $\{y \mid y\neq \frac{3}{2}\}$

VA: $x=-2$

HA: $y=\frac{3}{2}$

OA: —

Holes: —



D: $\{x \mid x \neq -3, 3\}$

R: $\mathbb{R}$

VA: $x=-3$, $x=3$

HA: —

OA: $y=x$

Holes: —

$$f(x)=\frac{x^2-2x-3}{x^2-1}$$

$$f(x)=\frac{(x-3)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$f(x)=\frac{x-3}{x-1}$$ degree: 1 / degree: 1

$$f(-1)=\frac{-1-3}{-1-1}$$

$$f(-1)=\frac{-4}{-2}$$

$$f(-1)=2$$

D: $\{x \mid x \neq -1, 1\}$

VA: $x=1$

HA: $y=1$

*Holes: $(-1, 2)$

cancelled zero

$$f(x)=\frac{x^2-4}{x^2+2x-8}$$

$$f(x)=\frac{(x+2)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$$

$$f(x)=\frac{x+2}{x+4}$$ degree: 1 / degree: 1

$$f(2)=\frac{2+2}{2+4}$$

$$=\frac{4}{6}$$

$$f(2)=\frac{2}{3}$$

D: $\{x \mid x \neq -4, 2\}$

VA: $x=-4$

HA: $y=1$

*Holes: $\left(2, \frac{2}{3}\right)$

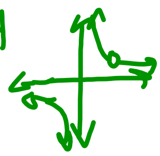
cancelled zero

Asymptotes of Rational Functions

- Vertical – graph will never touch or cross "undefined"
  1) put the function in lowest terms by factoring
  2) locate zeros of the denominator
  3) vertical asymptotes: x = "zero"
  4) graph will have a hole at "canceled zeros"

$$f(x)=\frac{(x+2)\cancel{(x-3)}}{(x+4)\cancel{(x-3)}}$$

$$f(x)=\frac{x+2}{x+4}$$



D: $\{x \mid x \neq -4, 3\}$

VA: $x=-4$

Hole: $\left(3, \frac{5}{7}\right)$

$$f(3)=\frac{3+2}{3+4}$$

Asymptotes cont. "end behavior"

- Horizontal – graph may touch or cross
  1) if the degree of the numerator is less than the degree of the denominator, graph will have a horizontal asymptote at y = 0
  2) if the degree of the numerator is equal to the degree of the denominator, graph will have a horizontal asymptote at

     y = leading coefficient of top/leading coefficient of bottom

$$f(x)=\frac{3x+2}{x^2-4x-5}$$ degree: 1 / degree: 2

HA: $y=0$

$$f(x)=\frac{4x-5}{6x-7}$$ degree: 1 / degree: 1

HA: $y=\frac{2}{3}$

Asymptotes cont. "end behavior"

- Oblique – graph may cross or touch
  1) if the degree of the numerator is one bigger than the degree of the denominator, graph will have an oblique asymptote

** To find the oblique asymptote, use long division and asymptote will be y = mx + b

$$f(x)=\frac{4x^2-5}{x-3}$$ degree: 2 / degree: 1

OA: $y=4x+12$

$$\begin{array}{r} 4x+12 \\ x-3\overline{)4x^2+0x-5} \\ -4x^2+12x \\ \hline 12x-5 \\ -12x+36 \\ \hline 31 \end{array}$$

Other End Behavior:

- if the degree of the numerator is more than one bigger than the degree of the denominator, the end behavior will be determined by long division and the type of quotient that occurs

$$f(x)=\frac{x^4+1}{x^2}$$

$$\begin{array}{r} x^2 \\ x^2 \overline{\smash{\big)}\, x^4+0x^3+0x^2+0x+1} \\ \underline{-x^4} \\ 0 \end{array}$$

$\boxed{\text{EB: } y=x^2}$

* If the degree of the top is more than one bigger than the degree of the bottom, the function will still have "end behavior," but it will not be linear.

Important Note

- The graph of a rational function either has one horizontal asymptote or one oblique asymptote or else has no horizontal or no oblique asymptote.
- WHY??

A graph can only have one end behavior.

$$f(x)=\frac{x^2}{x^4-1}$$

$$f(x)=\frac{x^2}{(x^2+1)(x^2-1)}$$

$$f(x)=\frac{x^2}{(x^2+1)(x+1)(x-1)}$$

D: $\{x \mid x \neq -1, 1\}$

VA: $x=-1$, $x=1$

HA: $y=0$

OA: —

Holes: —

$$f(x)=\frac{3x^2-2}{4x^2-1}$$

$$f(x)=\frac{3x^2-2}{(2x+1)(2x-1)}$$

D: $\{x \mid x \neq -\frac{1}{2}, \frac{1}{2}\}$

VA: $x=-\frac{1}{2}$, $x=\frac{1}{2}$

HA: $y=\frac{3}{4}$

OA: —

Holes: —

$$f(x)=\frac{6x^2+3}{2x-1}$$

$$\begin{array}{r} 3x+\frac{3}{2} \\ 2x-1\,\overline{)\,6x^2+0x+3} \\ -6x^2+3x \\ \hline 3x+3 \\ -3x+\frac{3}{2} \\ \hline \frac{9}{2} \end{array}$$

$\frac{9}{2} \leftarrow$ remainder

D: $\{x \mid x \neq \frac{1}{2}\}$

VA: $x=\frac{1}{2}$

HA: —

OA: $y=3x+\frac{3}{2}$

Holes: —

$$f(x)=\frac{x^2+5x+4}{x^2+6x+8}$$

$$f(x)=\frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x+2)}$$

$$f(x)=\frac{1x+1}{1x+2}$$

$$f(-4)=\frac{-4+1}{-4+2}$$

$$f(-4)=\frac{3}{2}$$

D: $\{x \mid x \neq -4, -2\}$

VA: $x=-2$

HA: $y=1$

OA: —

Holes: $(-4, \frac{3}{2})$

$$f(x)=\frac{x^4+1}{x^2-4}$$

$$f(x)=\frac{x^4+1}{(x+2)(x-2)}$$

$$\begin{array}{r|l} & x^2+4 \\ x^2-4 & x^4+0x^3+0x^2+0x+1 \\ & -x^4 \quad +4x^2 \\ \hline & \quad 4x^2+0x+1 \\ & -4x^2 \quad +16 \\ \hline & \quad 17 \end{array}$$

D: $\{x \mid x \neq -2, 2\}$

VA: $x=-2, x=2$

HA: —

OA: —

EB: $y=x^2+4$

Holes: —

Assignment: p.236 #2-18even, 24, 26, 30 and 32-40even
* write the problems and/or draw the given graph

#2-12 even - find domain

#14-18 even - label information from graph (draw graph)

#24,26,30 - graph using transformations

#32-40 even - find and label VA, HA and/or OA