

Today's Plan:

Learning Target (standard): I will use the remainder theorem and factor to theorem to locate zeros of polynomials.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Division of Polynomials

$$1) 5x^2 - 8x + 2 - \frac{5}{5x-1}$$

$$2) x^2 - 9x + 9 - \frac{9}{7x-4}$$

$$3) 7k^2 + 8k + 10 + \frac{1}{4k+8}$$

$$4) v^2 + 10v - 6 + \frac{3}{9v+2}$$

$$5) n^3 - 7 + \frac{3}{n+7}$$

$$6) x^3 + 6 - \frac{3}{x-8}$$

$$7) n^3 - 9n^2 - 10n - 10 - \frac{10}{n-6}$$

$$8) x^3 + x^2 + x + 8 + \frac{4}{x-6}$$

Simplify:

$$\frac{3x^3 + 2x - 4}{2x^2 + 1}$$

$$2x^2 + 1 \overline{) 3x^3 + 0x^2 + 2x - 4}$$

$\frac{3x^3}{2x^2} = \frac{3x}{2}$
 $\underline{-3x^3} \quad \underline{+ \frac{3}{2}x}$
 $\frac{1}{2}x - 4$

$$= \frac{3}{2}x + \frac{\frac{1}{2}x - 4}{2x^2 + 1}$$

$$= \frac{3}{2}x + \frac{x - 8}{2(2x^2 + 1)}$$

$$\frac{\frac{1}{2}(x-8)}{2x^2+1}$$

$$\frac{x-8}{2(2x^2+1)}$$

Simplify:

$$\frac{x^3 - 8x - 5}{x + 3}$$

$$-3 \overline{) \begin{array}{cccc} 1 & 0 & -8 & -5 \\ \downarrow & -3 & 9 & -3 \\ \hline 1 & -3 & 1 & -8 \end{array}}$$

$$x^2 - 3x + 1 - \frac{8}{x+3}$$

Simplify:

$$\frac{9x+4}{2x-5}$$

$$2x-5 \overline{) \begin{array}{r} 9x+4 \\ -9x+\frac{45}{2} \\ \hline \frac{53}{2} \end{array}}$$

$\frac{9x}{2x}$

$$= \frac{9}{2} + \frac{53}{2(2x-5)}$$

Simplify:

$$\frac{-3x^5 - 6x^2 - 7}{2-x} = \frac{-3x^5 - 6x^2 - 7}{-x+2} = \frac{3x^5 + 6x^2 + 7}{x-2}$$

$$2 \overline{) \begin{array}{r} 3 \ 0 \ 0 \ 6 \ 0 \ 7 \\ 6 \ 12 \ 24 \ 60 \ 120 \\ \hline 3 \ 6 \ 12 \ 30 \ 60 \ 127 \end{array}}$$

$$3x^4 + 6x^3 + 12x^2 + 30x + 60 + \frac{127}{x-2}$$

Simplify:

$$\frac{2x^6 - 18x^4 + x^2 - 9}{x + 3}$$

$$\begin{array}{r|rrrrrrr} -3 & 2 & 0 & -18 & 0 & 1 & 0 & -9 \\ & & -6 & 18 & 0 & 0 & -3 & 9 \\ \hline & 2 & -6 & 0 & 0 & 1 & -3 & 0 \end{array}$$

$$2x^5 - 6x^4 + x - 3$$

Remainder Theorem

□ Let $f(x)$ be a polynomial function. If $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

□ Example: Find the remainder when $f(x) = x^3 - 4x^2 + 2x - 5$ is divided by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 2 & -5 \\ & & 3 & -3 & -3 \\ \hline & 1 & -1 & -1 & -8 \end{array} \text{ remainder}$$

$$\begin{aligned} f(3) &= (3)^3 - 4(3)^2 + 2(3) - 5 \\ &= 27 - 36 + 6 - 5 \end{aligned}$$

$$f(3) = -8$$

↖ remainder

Factor Theorem

□ Let $f(x)$ be a polynomial function.
Then $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$. *remainder is 0*

□ Example: Use the Factor Theorem to determine whether the function $f(x) = 2x^3 - x^2 + 2x - 3$ has the factor $x-1$ or $x+3$.

$$\begin{aligned} \textcircled{1} f(1) &= 2(1)^3 - (1)^2 + 2(1) - 3 \\ &= 2 - 1 + 2 - 3 \\ f(1) &= 0 \end{aligned}$$

∴ According to the Factor Theorem since $f(1) = 0$ and that is the remainder, then $x-1$ is a factor of $f(x)$.

∴ According to the Factor Theorem, $(x-1)$ is a factor of $f(x)$ because $f(1) = 0$ and that is the remainder after division.

$$\begin{aligned} f(-3) &= 2(-3)^3 - (-3)^2 + 2(-3) - 3 \\ &= -54 - 9 - 6 - 3 \\ f(-3) &= -72 \end{aligned}$$

∴ According to the Factor Theorem, $(x+3)$ is not a factor of $f(x)$ because $f(-3) = -72$ and that is the remainder after division.

Number Of Zeros "solutions & roots"

□ A polynomial function cannot have more zeros than its degree.

□ Example: If $f(x) = 2x + 1$, the maximum number of zeros it can have is 1.

$$\begin{aligned} f(x) &= 3x^4 + 5x^2 - 6x + 2 \\ \text{degree: } &4 \\ \text{MNZ: } &4 \end{aligned}$$

$$\begin{aligned} f(x) &= 2 \\ \text{degree: } &0 \\ \text{MNZ: } &0 \end{aligned}$$

$$\begin{aligned} f(x) &= 0 \\ \text{degree: } &\text{none} \\ \text{MNZ: } &\text{none} \end{aligned}$$

Rational Zeros Theorem

- For $\frac{p}{q}$ to be a zero of a polynomial function, p must be a factor of the constant term and q must be a factor of the leading coefficient.
- Example: Find the p 's and q 's for $f(x) = x^3 + 2x^2 - 5x - 6$.

* If $f(x)$ factors over the rational numbers, its factors will be in the list of $\frac{p}{q}$.

$$p: \pm 1, \pm 2, \pm 3, \pm 6 \text{ (factors of the } -6\text{)}$$

$$q: \pm 1 \text{ (factors of the } 1 \text{ on the } x^3\text{)}$$

$$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6 \text{ (each "p" divided by each "q")}$$

Assignment:

p.261 #2-34 even

* On #12-22, tell the degree and the maximum number of zeros (ignore the instructions in the book) *