

Today's Plan:

Learning Target (standard): I will review solving systems of equations using the graphing, substitution and elimination methods. I will review solving systems using Cramer's Rule and the matrix method. I will perform operations on matrices and simplify.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and review homework assignment in preparation for test

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Inverse Matrices:

$$\begin{array}{ll}
 1) A^{-1} = \begin{bmatrix} \frac{7}{8} & -\frac{1}{8} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix} & 3) A^{-1} = \begin{bmatrix} \frac{5}{9} & -\frac{1}{9} \\ \frac{1}{9} & -\frac{2}{9} \end{bmatrix} & 5) A^{-1} = \begin{bmatrix} \frac{6}{31} & -\frac{7}{31} \\ -\frac{1}{31} & \frac{4}{31} \end{bmatrix} \\
 2) A^{-1} = \begin{bmatrix} \frac{1}{6} & -\frac{1}{3} \\ -\frac{7}{18} & \frac{4}{9} \end{bmatrix} & 4) A^{-1} = \begin{bmatrix} \frac{1}{10} & \frac{1}{10} \\ \frac{2}{25} & -\frac{1}{50} \end{bmatrix} & 6) A^{-1} = \begin{bmatrix} \frac{5}{19} & \frac{7}{19} \\ \frac{2}{19} & -\frac{1}{19} \end{bmatrix} & 7) A^{-1} = \begin{bmatrix} -3 & \frac{7}{2} \\ -1 & 1 \end{bmatrix} \\
 & & & 8) \text{no inverse}
 \end{array}$$

Find the product, if it exists.

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2+3-1 & 1+9+0 \\ 4+0-4 & 2+0+0 \end{bmatrix}$$

$$(2 \times 3)(3 \times 2) = 2 \times 2$$

$$= \begin{bmatrix} 4 & 10 \\ 0 & 2 \end{bmatrix}$$

Find the inverse matrix, if it exists. Verify.

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \quad \mathcal{D} = 10 - 12$$

$$\mathcal{D} = -2 \quad \checkmark$$

$$\begin{bmatrix} 2 & 4 & | & 1 & 0 \\ 3 & 5 & | & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & | & \frac{1}{2} & 0 \\ 3 & 5 & | & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & | & \frac{1}{2} & 0 \\ 0 & -1 & | & -\frac{3}{2} & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & | & \frac{1}{2} & 0 \\ 0 & 1 & | & \frac{3}{2} & -1 \\ 0 & -2 & | & -3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & -\frac{5}{2} & 2 \\ 0 & 1 & | & \frac{3}{2} & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{bmatrix} = \begin{bmatrix} -5+6 & 4-4 \\ -\frac{15}{2}+\frac{15}{2} & 6-5 \end{bmatrix}$$

$$(2 \times 2)(2 \times 2) = 2 \times 2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \checkmark$$

Find the determinant.

$$\begin{vmatrix} 2 & -3 & 4 \\ 1 & -2 & 3 \\ -3 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ -3 & 2 \end{vmatrix}$$

$$= 2(-2-6) + 3(1+9) + 4(2-6)$$

$$= -16 + 30 - 16$$

$$\boxed{D = -2}$$

Find the determinant.

$$\begin{vmatrix} 3 & -2 & -1 \\ -1 & 2 & -4 \\ 3 & -2 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & -4 \\ -2 & 2 \end{vmatrix} + 2 \begin{vmatrix} -1 & -4 \\ 3 & 2 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= 3(4-8) + 2(-2+12) - 1(2-6)$$

$$= -12 + 20 + 4$$

$$\boxed{D = 12}$$

Find the inverse matrix, if it exists. Verify.

$$\begin{bmatrix} -1 & 3 \\ -2 & 6 \end{bmatrix}$$

$$D = -6 + 6$$

$$D = 0$$

no inverse

Find the inverse matrix, if it exists. Verify.

$$\begin{bmatrix} 4 & 2 \\ -3 & 2 \end{bmatrix}$$

$$D = 8 + 6$$

$$D = 14$$

$$\begin{bmatrix} 4 & 2 & : & 1 & 0 \\ -3 & 2 & : & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & : & \frac{1}{4} & 0 \\ -3 & 2 & : & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{2} & : & \frac{1}{4} & 0 \\ 0 & \frac{7}{2} & : & \frac{3}{4} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & : & \frac{1}{4} & 0 \\ 0 & 1 & : & \frac{3}{14} & \frac{2}{7} \\ 0 & -\frac{1}{2} & : & -\frac{3}{28} & -\frac{1}{7} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & : & \frac{1}{7} & -\frac{1}{7} \\ 0 & 1 & : & \frac{3}{14} & \frac{2}{7} \end{bmatrix} \quad A^{-1} = \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ \frac{3}{14} & \frac{2}{7} \end{bmatrix}$$

Verify:

$$A^{-1} = \frac{1}{14} \begin{bmatrix} 2 & -2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ \frac{3}{14} & \frac{2}{7} \end{bmatrix} \checkmark$$

Verify Inverse (2x2):

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{D} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{d}{D} & \frac{-b}{D} \\ \frac{-c}{D} & \frac{a}{D} \end{bmatrix}$$

$$3y - 6 = 5x \quad \text{Matrix Method}$$

$$7y - 22 = 9x$$

$$5x - 3y = -6$$

$$9x - 7y = -22$$

$$\begin{bmatrix} 5 & -3 & : & -6 \\ 9 & -7 & : & -22 \end{bmatrix} \xrightarrow{\substack{-9 \\ \text{multiply}}} \begin{bmatrix} 1 & -7 & : & -22 \\ 9 & -7 & : & -22 \end{bmatrix} \xrightarrow{\substack{-9 \\ \text{subtract}}} \begin{bmatrix} 1 & -7 & : & -22 \\ 0 & 0 & : & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -\frac{3}{5} & : & -\frac{6}{5} \\ 0 & 1 & : & 7 \end{bmatrix} \quad \begin{array}{l} x - \frac{3}{5}y = -\frac{6}{5} \\ y = 7 \end{array} \quad \begin{array}{l} x - \frac{21}{5} = -\frac{6}{5} \\ x = 3 \end{array}$$

independent
(3, 7)

$$\begin{aligned} -3x + 4y &= -1 \\ 6x - 5y &= -13 \end{aligned} \quad \text{Cramer's Rule}$$

$$D = \begin{vmatrix} x & y \\ -3 & 4 \\ 6 & -5 \end{vmatrix} = 15 - 24$$

$$D = -9$$

$$D_x = \begin{vmatrix} -1 & 4 \\ -13 & -5 \end{vmatrix} = 5 + 52$$

$$D_x = 57$$

$$x = \frac{D_x}{D} = \frac{57}{-9} = -\frac{19}{3}$$

$$D_y = \begin{vmatrix} -3 & -1 \\ 6 & -13 \end{vmatrix} = 39 + 6$$

$$D_y = 45$$

$$y = \frac{D_y}{D} = \frac{45}{-9} = -5$$

independent
 $(-\frac{19}{3}, -5)$

$$5x - 2y = 4 \quad \text{Matrix Method}$$

$$x - 2y = -4$$

$$\begin{bmatrix} 1 & -2 & : & -4 \\ 5 & -2 & : & 4 \end{bmatrix} \xrightarrow{\substack{-5 \\ 10 \\ 20}} \begin{bmatrix} 1 & -2 & : & -4 \\ 0 & 8 & : & 24 \end{bmatrix} \xrightarrow{\substack{1 \\ 1}} \begin{bmatrix} 1 & -2 & : & -4 \\ 0 & 1 & : & 3 \end{bmatrix}$$

$$x - 2y = -4$$

$$y = 3$$

$$x - 6 = -4$$

$$x = 2$$

independent
 $(2, 3)$

Solve using the matrix method.

$$5x + 4y + 3z = -9$$

$$x - 2y + 2z = -6$$

$$x - y - z = 3$$

$$\begin{array}{c}
 \begin{matrix} -1 & 1 & 1 & -3 \end{matrix} \\
 \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 1 & -2 & 2 & : & -6 \\ 5 & 4 & 3 & : & -9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 0 & -1 & 3 & : & -9 \\ 5 & 4 & 3 & : & -9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 0 & -1 & 3 & : & -9 \\ 0 & 9 & 8 & : & -24 \end{bmatrix} \\
 \begin{matrix} 0 & -9 & 27 & -81 \end{matrix} \\
 \rightarrow \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 0 & 1 & -3 & : & 9 \\ 0 & 9 & 8 & : & -24 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 0 & 1 & -3 & : & 9 \\ 0 & 0 & 35 & : & -105 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -1 & : & 3 \\ 0 & 1 & -3 & : & 9 \\ 0 & 0 & 1 & : & -3 \end{bmatrix}
 \end{array}$$

Assignment:

Matrix Review #1-17