

Today's Plan:

Learning Target (standard): I will find the average velocity and instantaneous velocity of a particle moving along a curve.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Velocity Worksheet

1) changes direction at $t = \frac{13}{2}$

moves left: $t > \frac{13}{2}$

moves right: $0 < t < \frac{13}{2}$

$$v(t) = -2t + 13$$

2) changes direction at $t = 8$

moves left: $0 < t < 8$

moves right: $t > 8$

$$v(t) = 3t^2 - 24t$$

Velocity Worksheet

3) changes direction at $t = 3, 9$

moves left: $3 < t < 9$

moves right: $0 \leq t < 3, t > 9$

$$v(t) = 3t^2 - 36t + 81$$

4) changes direction at $t = 8$

moves left: $t > 8$

moves right: $0 < t < 8$

$$v(t) = -3t^2 + 24t$$

Velocity Worksheet

5) changes direction at $t = 4$

moves left: $t > 4$

moves right: $0 < t < 4$

$$v(t) = -2t + 8$$

6) changes direction at $t = 2$

moves left: $t > 2$

moves right: $0 < t < 2$

$$v(t) = -2t + 4$$

$$f(x) = 4x^3 + 2x - 5$$

$$m_{\tan(a, f(a))} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$m_{\tan(1,1)} = \lim_{h \rightarrow 0} \frac{[4(a+h)^3 + 2(a+h) - 5] - [4a^3 + 2a - 5]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4(a^3 + 3a^2h + 3ah^2 + h^3) + 2a + 2h - 5 - 4a^3 - 2a + 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{4a^3} + 12a^2h + 12ah^2 + 4h^3 + \cancel{2a} + 2h - \cancel{5} - \cancel{4a^3} - \cancel{2a} + \cancel{5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(12a^2 + 12ah + 4h^2 + 2)}{h}$$

$$= \lim_{h \rightarrow 0} (12a^2 + 12ah + 4h^2 + 2)$$

$$M_{\tan(a, f(a))} = 12a^2 + 2$$

$$M_{\tan(1,1)} = 12(1)^2 + 2$$

$$M_{\tan(1,1)} = 14$$

$$v(a) = 12a^2 + 2$$

left:

$$v(a) < 0$$

$$12a^2 + 2 < 0$$

right:

$$v(a) > 0$$

$$12a^2 + 2 > 0$$

stop:

$$v(a) = 0$$

$$12a^2 + 2 = 0$$

$f(x) = x^2 + 2x$ Explain the meaning of both results.

$$m_{\tan(a, f(a))} =$$

$$m_{\tan(2,8)} =$$

$$m_{\tan(a, f(a))} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(a+h)^2 + 2(a+h)] - [a^2 + 2a]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{a^2} + 2ah + h^2 + \cancel{2a} + 2h - \cancel{a^2} - \cancel{2a}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2a+h+2)}{h}$$

$$= \lim_{h \rightarrow 0} (2a+h+2)$$

$$m_{\tan(a, f(a))} = 2a + 2$$

$$m_{\tan(2,8)} = 2(2) + 2$$

$$m_{\tan(2,8)} = 6$$

The slope of the tangent line through $(a, f(a))$ to $f(x) = x^2 + 2x$ is $2a + 2$. The slope of the tangent line through $(2, 8)$ to $f(x) = x^2 + 2x$ is 6.

The position of a particle on a curve is given by $s(t) = t^2 - 6t$ feet. Find the average velocity on the time interval $[1, 1.2]$ seconds. Find the velocity at a seconds. What is the velocity at time 0 seconds and 4 seconds? Determine when the particle will move in the positive direction and the negative direction. At what time is the velocity 0 ft/sec?

$$\begin{aligned} \textcircled{1} \text{ ARVC} &= \text{avg vel} = \frac{s(1.2) - s(1)}{1.2 - 1} \\ &= \frac{[(1.2)^2 - 6(1.2)] - [1^2 - 6(1)]}{.2} \\ &= \frac{1.44 - 7.2 - 1 + 6}{.2} \\ &= \frac{-.76}{.2} \end{aligned}$$

$$\text{avg. vel.} = -3.8 \text{ ft/sec}$$

$$\begin{aligned} \textcircled{2} v(a) &= \lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[(a+h)^2 - 6(a+h)] - [a^2 - 6a]}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^2 + 2ah - 6a - 6h - a^2 + 6a}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2a - 6)}{h} \\ &= \lim_{h \rightarrow 0} (2a - 6) \end{aligned}$$

$$v(a) = 2a - 6 \text{ ft/sec}$$

$$\begin{aligned} \textcircled{3} v(0) &= 2(0) - 6 \\ v(0) &= -6 \text{ ft/sec} \end{aligned}$$

$$\begin{aligned} v(4) &= 2(4) - 6 \\ v(4) &= 2 \text{ ft/sec} \end{aligned}$$

④ positive direction

$$v(a) > 0$$

$$2a - 6 > 0$$

$$2a > 6$$

$$a > 3 \text{ sec}$$

⑤ negative direction

$$v(a) < 0$$

$$2a - 6 < 0$$

$$2a < 6$$

$$a < 3 \text{ sec}$$

⑥ equal 0

$$v(a) = 0$$

$$2a - 6 = 0$$

$$2a = 6$$

$$a = 3 \text{ sec}$$

The position of a particle at any time t in seconds is given by $s(t) = 2t^3 - t^2 + 1$ in feet. Find the velocity of the particle at a seconds, 2 seconds and 3 seconds. When will the particle stop moving?

$$\begin{aligned} \textcircled{1} v(a) &= \lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2(a+h)^3 - (a+h)^2 + 1] - [2a^3 - a^2 + 1]}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(a^3 + 3a^2h + 3ah^2 + h^3) - (a^2 + 2ah + h^2) + 1 - 2a^3 + a^2 - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2a^3} + 6a^2h + 6ah^2 + 2h^3 - \cancel{a^2} - 2ah - h^2 + \cancel{1} - \cancel{2a^3} + \cancel{a^2} - \cancel{1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6a^2 + 6ah + 2h^2 - 2a - h)}{h} \\ &= \lim_{h \rightarrow 0} (6a^2 + 6ah + 2h^2 - 2a - h) \end{aligned}$$

$$v(a) = 6a^2 - 2a \text{ ft/sec}$$

$$\begin{aligned} \textcircled{2} v(2) &= 6(2)^2 - 2(2) \\ &= 24 - 4 \\ v(2) &= 20 \text{ ft/sec} \end{aligned}$$

④ stops when $v(a) = 0$

$$6a^2 - 2a = 0$$

$$2a(3a - 1) = 0$$

$$a = 0, \frac{1}{3} \text{ sec}$$

$$\begin{aligned} v(3) &= 6(3)^2 - 2(3) \\ &= 54 - 6 \\ v(3) &= 48 \text{ ft/sec} \end{aligned}$$

Assignment:

p.89 #5,7,8